addresses stressing the importance of the meeting. Prof. Gonseth then introduced and explained the subject to be discussed.

At the meetings the following six papers were discussed : "The Significance of the Skolem-Löwenhein Theorem" (Prof. Th. Skolem) ; "General Analysis of the Question of Fundamentals" (Prof. M. Fréchet) ; "Logic and the Question of Fundamentals" (Prof. J. Łukasiewicz) ; "Controversies Regarding the Theory of 'Ensembles' and the Question of Fundamentals" (Prof. H. Lebesgue) ; "The Role of the Axiom of Choice, and the Hypothesis of Continuity in Mathematics" (Prof. W. Sierpinski) ; "The Present Position in regard to Hilbertism" (Prof. Bernays).

At the end of the meeting Prof. Gonseth summed up the general conclusions to be drawn from the discussions, and expressed his thanks to the Institute of Intellectual Co-operation for its help and collaboration.

The texts of the various papers submitted and an account of the discussions will appear in the fourth volume of the new Scientific Series being published by the Institute.

Molecular Structure

A DISCUSSION on molecular structure was held by the London Mathematical Society on January 19 in the lecture room of the Royal Astronomical Society, the president, Prof. E. A. Milne, being in the chair. The discussion was opened by Dr. W. G. Penney ; Dr. H. A. Jahn, Dr. C. A. Coulson and Mr. G. J. Kynch made further contributions, and Prof. J. E. Lennard-Jones summarized the discussion.

Dr. W. G. Penney (Imperial College, London) said that the nuclei of a polyatomic molecule in its most stable state are practically at rest and that the electrons move about in a highly complicated, but regular, way. If the nuclei are slightly disturbed, they vibrate, and their motion can be described in terms of a number of normal modes. Such vibrations can be detected in the infra-red spectrum.

Regular tetrahedron.

Group T_d.

Vibrations :

$A_1 + E + 2F_2$.

4 frequencies.

5 general force constants. F_2

Let $E(x_1, x_2 \ldots)$ be the least energy required to separate a molecule into electrons and nuclei, when the nuclei are stationary at $x_1, x_2 \ldots$. Then, in principle, $E(x_1, x_2 \ldots)$ may be found as the eigen values of the Schrödinger differential equation

$$(H - E)\, \psi = 0 \qquad (1)$$

The energy of formation of the molecule from atoms is

$$W(x_1, x_2 \ldots) = E(x_1, x_2 \ldots) - \Sigma E_a,$$

where ΣE_a is the sum of the energies of formation of the separate atoms from nuclei and electrons. The equations for the E_a are similar to (1). Since W is very small compared with ΣE_a, it is possible to calculate W by perturbation methods. Thus, ψ is expanded in terms of the ψ_a, the Schrödinger functions of the separate atoms. Substituting this in (1), leads to

$$[H^1 - W(x_1, x_2 \ldots)]\, \Sigma \alpha_a \psi_a = 0.$$

Multiplying by the ψ_a in succession, and integrating over the configurational space, leads to a set of equations in the α_a. Eliminating the α_a gives a determinantal equation for $W(x_1, x_2 \ldots)$. Minimizing with respect to $x_1, x_2 \ldots$ gives the equilibrium configuration and the heat of formation ; expanding about the minimal configuration leads to the frequencies of the normal modes.

Detailed considerations of the function $W(x_1, x_2 \ldots)$ show, for example, that if four monovalent atoms are combined with a carbon atom, they are distributed around the carbon atom in the directions of the corners of a regular tetrahedron. If the carbon atom is linked directly to only three atoms, so that one of the bonds is double, the system is co-planar. When the carbon atom is linked to only two atoms, so that both bonds are double, or one is triple and the other single, the system is linear.

Little progress has been made in calculating the energies of formation and the vibration frequencies of molecules in excited states, and the description of their absorption spectra is therefore at present only qualitative. A similar, but simpler, problem is that of interpreting the absorption spectra of rare earth crystals. The spectrum is that of an inner group of electrons of the metal ion, and the energy levels are very near those of the free ion.

Dr. H. A. Jahn (Davy Faraday Laboratory, Royal Institution) discussed the vibrational potential energy constants of polyatomic molecules. He showed how, starting from a given equilibrium configuration of a polyatomic molecule and making use of the invariance of the vibrational potential energy with respect to all the operations of the group of symmetry of the equilibrium configuration, the most general form of the elastic potential can be determined.

Mathematically, the invariance of the vibrational potential energy is equivalent to the fact that the matrix of its coefficients must commute with all those matrices which show how the normal displacements transform under the symmetry operations of

the group. Consequently, by choosing a complete set of vibrational symmetry co-ordinates which transform according to irreducible representations of the group of symmetry, there may be obtained at once, by application of Schur's Lemma, the most general form of the matrix of the potential energy.

This was illustrated by the example of the methane molecule.

Thus, if the methane molecule is a regular tetrahedron, the potential energy has five general force constants and the molecule has four different vibrational frequencies. Similarly, if the molecule has the form of a square pyramid, the group is C_{4v}, the vibrations are $2A_1 + B_1 + 2B_2 + 2E_1$, there are seven different frequencies and ten general force constants. Experimentally four frequencies are found, and the molecule has the form of a regular tetrahedron.

Dr. C. A. Coulson (University College, Dundee) discussed the theory of the carbon-carbon linkages in some simple polyenes and aromatic molecules. Certain hydrocarbons can be adequately described by a single structural formula. Examples are ethane, where the linkage is a single bond, ethylene, where the linkage is a double bond, and acetylene, where the linkage is a triple bond. Other molecules, such as benzene, naphthalene and butadiene, are more complex, and in these cases the linkages must be considered to be of fractional order. General methods of calculating the orders of these linkages have been discovered. They evaluate the energy of a linkage by the approximation of molecular orbitals or of electron pairs, in terms of a single parameter which measures the difference of energy of the single and double bonds. Linear interpolation then gives the order.

From the experimental values of certain physical constants of the single, double and triple bonds, empirical curves may be constructed. These may be used to predict values for the linkages of intermediate order. Thus, the benzene linkage is of order 1·65, and the internuclear distance should be 1·39 A., in exact agreement with direct observation. Similarly, in naphthalene, there should be four types of linkage. Their orders, beginning at the central link and working outwards, are 1·52, 1·56, 1·76, 1·60. The corresponding lengths are 1·42, 1·40, 1·38, 1·40 A.

Mr. G. J. Kynch (Imperial College, London) discussed the relations between the magnetic properties, specific heat and absorption spectra of rare earth crystals. The deviations of the principal and mean susceptibilities from Curie's law, $\chi T = $ constant, are caused by an electric crystalline field which perturbs the magnetic ion. Calculations of the magnetic susceptibilities involve the determination of the lowest energy levels of the magnetic ion. These can be compared with those obtained from the measurements of the absorption spectra and the specific heat. Previously, it was thought that a fourth order cubic field would correlate all the data. This no longer seems possible, because of the considerable magnetic anisotropy of the crystals. A new interpretation of the levels found spectroscopically must be sought.

The British Rhætic Flora

PROF. T. M. HARRIS, with experience of the Greenland Rhætic flora behind him, has now studied the British Rhætic flora and, with the application of new methods, described with considerable detail some of the few plants from these rocks (The British Rhætic Flora, British Museum, 1938.) The main outcrop of Rhætic rocks extends from Dorset to north Yorkshire, and it is from these alone that plant remains have been obtained, though smaller outcrops also occur in Wales, Northern Ireland and Scotland. The conception of the Rhætic formation is that of a shallow sea, dotted with islands, and in this, slight changes in level isolated lagoons, which would soon become brackish or freshwater lakes. It is probably in such lakes that the plants grew. The plant-containing beds occur as strata between those containing fish and other marine remains, suggesting temporary incursions of the sea into the lagoon region. The Bristol locality seems to have supplied most of the specimens.

A new method of examination of surface features by reflected light on specimens moistened with paraffin oil has contributed materially to the success of the investigation. The usual methods of rock sections and cellulose nitrate films have also been used ; but are difficult with this material.

A common plant is the alga *Botryococcus braunii*, recognizable as colonies, terra-cotta in colour and with characteristic resistance to $HNO_3 + KClO_3$. It is probable that this fossil is the same plant which has been described under various names from Ordovician, Carboniferous, Permian, Rhætic, Jurassic and other rocks up to the present day.

Another alga has close resemblance to *Cosmarium*; but owing to failure to recognize sculpturing of the wall, it is provisionally distinguished as *Stenixys cosmarioides*.

Perhaps the most interesting plant and one described in amazing detail is *Naiadita lanceolata*, an undoubted bryophyte. Earlier records of Bryophyta are very scanty and only allow the statement that liverworts, probably belonging to the Jungermanniales, extend back to the Carboniferous, whilst records of mosses are more doubtful.

Naiadota is now described with details of stem and leaf form and structure, rhizoids, gemmæ (free and possibly in gemmæ cups, and development of new plantules from gemmæ), archegonia, sporophytes, perianth and spores. The plant is thus sufficiently known to compare with present-day genera. The radial construction and spiral phyllotaxis (3/8) are moss-like features ; but the majority of the characters place it in the Hepaticæ, and provisionally as nearest to the Riellaceæ of the Sphærocarpales.

Prof. Harris discusses the conceptions of evolutionary trend in Bryophyta and regards *Naiadita* as supporting rather the view of a reduction series than an up-grade series.

One other plant, *Hepaticites solenotus*, appears to be a hepatic resembling *Aneura*.

Certain other fossils, previously described as Rhætic, seem more correctly to be Liassic.