be defined relatively to the *Hohlraum* is: $dz^1 = \frac{8\pi}{c^3} V.G(p)\,p^2\,dp$, where $G(p) \sim 1$ if $p < b$, and $G(p) \to 0$ if $p \to \infty$, and $b = h/r_0$ is a critical value of the impulse. It means that a Lorentz observer has no possibility of distinguishing between states belonging to an assembly of $n(p) = G^1(p)$ neighbouring states. (A discussion of the measure of $p$ by means of the Compton effect shows that the possibilities offered by this measure are not incompatible with the existence of the indeterminacy here examined.) For example, if a particle with $p > b$ produces showers by collision with the walls, the usual eigenstates become energetically connected and consequently indistinguishable. Considering the correlation between the quantum states and the cells of volume $h^3$ in the phase-space, we can say that it is impossible to distinguish experimentally elements of an assembly of $n(p)$ neighbouring cells, and thus it is necessary to consider such an assembly as constituting a unique quantum cell in the new theory. We assume, therefore, that the observables, impulse $p_x$ and co-ordinate $x$, relative to a reference frame individualized by the measuring apparatus, satisfy the commutation relation of the type:

$$\Delta p_x \, \Delta x \gtrsim h \cdot f(p_x),$$

where $f(p_x) \sim 1$ if $|p_x| < b$ and $f(p_x) \to \infty$ if $|p_x| \to \infty$.

Recently I have shown that it is possible to build an example of representatives of these new states and observables satisfying the following rules: the number of states for a unit impulse interval has a maximum for $p \sim b$; the total number of states is finite; the representatives of the states corresponding to the impulse-operator are not orthogonal; the orthogonality is approximately satisfied only for eigen values $p < b$.

In order to satisfy the claim of relativistic invariance, it is possible to substitute systematically for the impulses the modulus of the difference between two 4-vectors $p_\nu$ referred to an initial and a final state respectively. The consideration of the reference frame individualized by the measuring apparatus in this formulation of the theory gives results of great importance.

The most important consequences of the modified algebra of states and observables will be discussed in detail elsewhere. Let us confine ourselves to some remarks regarding the possible origin of Heisenberg's explosion-showers. According to the present quantum theory, the simultaneous production of many particles in a single quantum process is very improbable because it corresponds to a high-order process of the perturbation theory. In a first order transition the selection rules, derived in the case of photons from the orthogonality and peculiar properties of the eigenfunctions of a harmonic oscillator, forbid transitions with emission and absorption of more than one photon. The representatives of the new quantum states corresponding to $p > b$ are not orthogonal and differ sensibly from the usual representatives. Therefore the first order process with simultaneous emission of many particles becomes probable, with the same order of probability as the single particle transition.

A more detailed discussion shows that in the barycentric frame of two colliding particles the impulses of secondaries have their probable values $\sim b$ because the great majority of quantum states is condensed in the region $p \sim b$. In another reference frame, in which the barycentre is moving with an ultra-relativistic velocity, nearly all secondaries are projected, with $p \gg b$, within a small solid angle (as in the hard showers of Bothe). The existence of a lower limit of measurable lengths follows also from the assumption discussed above.

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## Indeterminacy and Electron Spin

SINCE a dipole of magnetic moment $M$ in a magnetic field of intensity $H$ has a potential energy $-M.H$, two free electrons in magnetized iron with magnetic axes respectively parallel and anti-parallel to the direction of magnetization should differ in potential energy by $Heh/2\pi mc$, where $H$ is the effective field intensity acting on a free electron in the iron. It may be of some interest to see whether this difference in energy could be detected in an ideal experiment, for example, by a splitting under magnetization of the photo-electric threshold of iron, or whether the quantum indeterminacy prevents the resolution, as Bohr has shown it must in a Stern-Gerlach experiment on a beam of free electrons[1].

Let the field be assumed parallel to a boundary of the iron. At the photo-electric threshold those electrons will just escape which reach the boundary with a velocity $v$ normal to the surface, the work function being given by

$$W = \tfrac{1}{2} mv^2.$$

The path of these electrons in the iron will be an arc in a plane normal to the direction of magnetization and having a radius given by

$$\rho = mvc/He.$$

They must therefore have received the kinetic energy $W$ at some point on a semi-circle in the iron, and their initial angular position is thus determined within the angle $\pi$. Canonically conjugate with the angular co-ordinate is the angular momentum given by

$$mv\rho = m^2v^2c/eH = W \cdot 2mc/eH.$$

The indeterminacy in the angular co-ordinate being $\pi$, we have:

$$\pi \cdot \triangle(mv\rho) > h$$

whence

$$\triangle W > Heh/2\pi mc.$$

But this is just the difference in energy that was to be resolved, and the resolution is thus impossible.

If the direction of magnetization is normal to the boundary, the conclusion is the same. The argument in this case resembles that for the Stern-Gerlach experiment in that it depends on the divergence equation of the magnetic field.

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[1] See Mott, N. F., *Proc. Roy. Soc.*, A, **124**, 425 (1929).