the faint satellite of Saturn, of which Prof. W. H. Pickering reported the discovery in 1905, but which has not since been observed, is the missing body.

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Methods of Estimating Mental Factors

MR. M. S. BARTLETT has proposed¹ formulae for the estimation of mental factors as alternatives to formulae I have given². My formulae were arrived at by the ordinary regression method. Bartlett's estimates and the regression estimates attain different ends, and it is agreed that each method is correct in the right place. The regression estimates minimize the squares of the discrepancies between the estimates and the true values, summed over the population of persons. Bartlett's estimates minimize the squares of a man's *specific* factors, summed over the tests. If the scores expressed in matrix notation as linear functions of the true factors are

$$Z = MF = M_0F_0 + M_1F_1$$

the two sets of estimates, for the common factors only, are

$$\hat{F}_0 = M_0'(MM')^{-1}Z = M_0'R^{-1}Z \text{ (regression estimates)}$$

$$\tilde{F}_0 = (M_0'M_1^{-2}M_0)^{-1}M_0'M_1^{-2}Z \text{ (Bartlett's estimates).}$$

The regression method has the quality that if factors estimated by it are inserted in the specification equation which defines a vocation in terms of factors, the result is the same as a direct regression estimate of vocational ability from the tests.

If estimates, by either method, of all the factors including specifics are inserted into the specifications MF of the test scores, they reproduce the latter exactly. If, however, only the common factors are estimated and used, Bartlett's reproduce the scores better than the regression estimates do. In such a situation, however, the regression method would again *estimate* the original scores from the estimated common factors, and it is the object of this note to point out that if this is done, the results are identical with those produced by Bartlett's method.

If we estimate the quantities Z from the $\hat{F}_0$ only, by the ordinary methods of regression, we get, bearing in mind that the $\tilde{F}_0$ are neither normalized nor uncorrelated,

$$\hat{Z} = M_0(M_0'R^{-1}M_0)^{-1}M_0'R^{-1}Z = SZ \text{ say.}$$

Whether we simply insert Bartlett's estimates in M_0F_0 , or make a regression estimate of the scores from the $\tilde{F}_0$, we reach the result

$$\hat{Z} = M_0(M_0'M_1^{-2}M_0)^{-1}M_0'M_1^{-2}Z = EZ \text{ say,}$$

and we see that the matrices S and E are identical if we use the relations

$$R = M_0M_0' + M_1^2$$

and

$$(M_0'R^{-1}M_0)^{-1} = I + (M_0'M_1^{-2}M_0)^{-1}.$$

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¹ Brit. J. Psych., 28, 99 (1937).

² (American) J. Educ. Psych., 27, 41 (1936).

Blood Group Inheritance

THE unusual blood group proportions among the Soussoux in French Guinea, found by J. S. de Goldfiem¹ and quoted by R. Elson Dew², are inexplicable as a stable distribution on the accepted triple allelomorph theory of blood group inheritance. They are, however, quite consistent with the quadruple allelomorph theory proposed by us³, if it be assumed that the fourth allelomorph, the AB gene, while rare in most human populations, is the predominant one among the Soussoux. The hypothesis could be tested as follows.

Denoting the four allelomorphic genes R, A, B, D (that is, ab, Ab, aB, AB in our previous notation), let their frequencies be $\gamma, \alpha, \beta, \delta$; and let the frequencies of the blood groups O, A, B, AB be r, p, q, s . Then for a stable distribution,

$$\gamma^2 = r, \alpha^2 + 2\alpha\gamma = p, \beta^2 + 2\beta\gamma = q,$$

$$\delta^2 + 2\alpha\delta + 2\beta\delta + 2\gamma\delta + 2\alpha\beta = s.$$

$$\text{Hence } \alpha = \sqrt{p+r} - \gamma, \beta = \sqrt{q+r} - \gamma,$$

$$\gamma = \sqrt{r}, \delta = 1 - \sqrt{p+r} - \sqrt{q+r} + \gamma.$$

(For the triple allelomorph theory, there is the additional equation $\delta = 0$.) The data in question are

$$r = 0.01, p = 0.01, q = 0.09, s = 0.89,$$

whence $\gamma = 0.100, \alpha = 0.041, \beta = 0.216, \delta = 0.642$.

The gene distribution would therefore be

10 per cent R + 4 per cent A + 22 per cent B + 64 per cent D .

Calculation on this basis gives the following results for the offspring of two parents, one being of group AB (in other cases the usual theory applies).

$$AB \times AB = 0.75 (AB) + 0.025 (A + 3AB) + 0.20 (B + 3AB) + 0.02 (O + 3AB) + 0.005 [A + B + 2AB],$$

$$A \times AB = 0.52 (AB) + 0.26 (B + 3AB) + 0.12 (O + A + 2AB) + 0.085 [A + AB] + 0.015 [2A + B + AB],$$

$$B \times AB = 0.49 (AB) + 0.03 (A + 3AB) + 0.07 (O + B + 2AB) + 0.40 [B + AB] + 0.01 [A + 2B + AB],$$

$$O \times AB = 0.46 (AB) + 0.06 (A + AB) + 0.31 (B + AB) + 0.15 (O + AB) + 0.02 [A + B].$$

The notation means that in 75 per cent of families with parents $AB \times AB$ the offspring will be all AB , in 2½ per cent they will be A and AB , expected in the ratio 1 : 3, and so on. On the usual theory, only the family types in square brackets should occur.

It will be seen that the majority of offspring of $O \times AB$ will be either O or AB , that is, anomalies on the usual theory; and a few such anomalies appear in the other three cases. The fourth allelomorph was in fact suggested in order to account for just such anomalies in ordinary populations. Other suggestions which have been made to account for them are *mutation* by various investigators, *crossing-over* by Bauer⁴, *trisomy* by Haldane⁵. An examination of blood group pedigrees among the Soussoux would perhaps settle the matter.

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¹ C.R. Soc. Biol., 123, 391 (1936).

² NATURE, 140, 1066 (Dec. 18, 1937).

³ NATURE, 136, 297 (Aug. 24, 1935); 646 (Oct. 19, 1935).

⁴ Klin. Wochenschr., 7, 1588 (1928).

⁵ NATURE, 136, 432 (Sept. 14, 1935).