A Method of Determining Stellar Rotation*

WHILE rotational speeds of stars can be estimated from photographs of stellar spectra, the actual deduction of these speeds and the demonstration that the star is rotating depend on rather abstruse mathematical calculation which, however, leads to a comparatively simple working routine. A knowledge of stellar rotational speeds, while obviously worth determining in any event, has several immediate applications, such as assessing the age of the stellar system, and is especially important in discussing the state of affairs in stellar atmospheres, a high rotational speed tending to offset gravity and promote ionization. If the rotational speed of a star is determinable, then the 'proper' spectrum of the star can be deduced; that is, the light emitted by any element of the star's surface, as distinct from the 'integrated' spectrum directly observed. Owing to the minuteness of a stellar image in a telescope, the light from the whole of the star's surface passes into the spectroscope, and the rotational speed of a star cannot be measured as can that of the sun by observing the Doppler displacement of the Fraunhofer lines from one limb as compared with the other.

Prof. Carroll's method depends on the fact that the observed shape of a stellar absorption line (that is, the graph of intensity plotted against wavelength) has certain peculiarities if the star is rotating.

If a star be rotating, then different elements of area on the surface of the star are moving towards or away from the observer with velocities in the line of sight which depend on their position on the stellar surface and the rotational speed of the star. Thus, the spectra from the different elements do not simply superpose, as they would in a non-rotating star, but are displaced somewhat with respect to each other, thus resulting in a shallow broadened line. The final observed line shape therefore depends on two factors, namely, (a) the shape of the line emitted by any element of the star and (b) the rotational displacement. The first factor is quite unknown—previous methods of estimating stellar rotational speeds, however, depended on assuming this to be known—but the second factor can be detected by the new method.

The equations connecting the 'proper' shape $\mathfrak{I}(\mathfrak{T})$ of a line and the observed shape $\mathfrak{O}(\mathfrak{T})$ are

$$\mathfrak{O}(\mathfrak{T}) = \frac{1}{a}\int_{-1}^{+1} \mathfrak{I}(\mathfrak{T} + \beta t)\, g(t)\, dt$$

where a is a numerical factor and equal to $\int_{-1}^{+1} g(t)dt$, and $g(t)$ is a function that depends on the way the brightness of the star varies over its disk, and β is the rotational speed of the star.

The solution of this equation is[1]:

$$\mathfrak{I}(\mathfrak{T}) = \frac{1}{2\pi\iota}\int_{0}^{\infty} \frac{e^{-ux}}{G(\beta x)} \int_{c} \mathfrak{O}(z) e^{-zx} dz dx,$$

where $$G(u) = \frac{1}{a}\int_{-1}^{+1} e^{-ut}\, g(t) dt\,;$$

but this is unusable as it stands as $\mathfrak{O}(z)$ is to be integrated over a certain contour in the complex domain and $\mathfrak{O}(z)$ is only known for real values of z. Analytical investigation shows a simple way out. We form the real part of the Fourier transform of $\mathfrak{O}(\mathfrak{T})$, namely, $f(u) = \int_{-\infty}^{\infty} \mathfrak{O}(\mathfrak{T}) \cos u\mathfrak{T}\, d\mathfrak{T}$, which is easily done and plotted as a graph of $f(u)$ against u. It appears that, if the star be rotating, $f(u)$ must vanish (have zeros) at certain values of the product $u\beta$. Now we find, by plotting $f(u)$, the values of u for which $f(u)$ vanishes and hence by simple division the value of β.

It is, of course, possible for $f(u)$ to vanish for other reasons; the importance of the method lies in the fact that, if $f(u)$ has several zeros, they must give (or some of them), if the star is rotating, consistent values of β. A further test is that the values of β from different lines must be the same. Thus, if several zeros from several lines all yield substantially the same value for β, we can feel with some confidence that the observed line shapes are really produced by rotational broadening. As the theoretically calculated values of $u\beta$ for which $f(u)$ vanishes depend somewhat on the law of darkening involved in $g(t)$, it is theoretically possible to test this law as well. Having found β, a further simple computation (forming $\mathfrak{I}(\mathfrak{T})$ from its transform) yields the 'proper' shape of the emitted line.

A similar method can be applied to the problem of stellar expansion.

In interpretation of this procedure, it was explained that any arbitrary function of the type considered might be represented by a 'spectrum' of harmonic constituents (Fourier's integral theorem), the result obtained meaning that, if the observed line shape is due to rotation, certain of these harmonic constituents must necessarily be absent. Practical tests for this give the information (a) whether the star is rotating, (b) the speed of rotation, that is, the equatorial velocity in the line of sight, and (c) thus enabling the 'proper' shape of the lines in the stellar spectrum to be found.

Unfortunately, the lines to be examined are, if the rotational speed be high enough to be readily determinable, wide, faint, shallow lines, and the technique of stellar photographic spectrophotometry is strained to the uttermost in providing sufficiently accurate measurements of line shape. Prof. F. J. M. Stratton, of the Solar Physics Observatory, Cambridge, is collaborating with the Natural Philosophy Department of the University of Aberdeen in attempting to produce sufficiently good observations, and very promising results are now being obtained.

In the case of one star, namely, Algol, independent measurements of rotational speed are available, and the direct measurements give 26 km./sec. as the equatorial line of sight velocity for this star, compared to 26 ± 3 km./sec. determined by Prof. Carroll's method, using an exceptionally well determined profile of the line Mg^{+} at 4481A. obtained by Struve at Yerkes Observatory.

* Summary of a lecture on "Rotating Stars" delivered before the Newcastle-upon-Tyne Astronomical Society at Armstrong College, on March 11, by Prof. J. A. Carroll.

[1] *Mon. Not. Roy. Ast. Soc.*, **93**. 478, 508, 680 (1933).