The Hexlet

DR. FRANK MORLEY has given in NATURE of January 9, p. 72, a very elegant proof of my hexlet theorem which incidentally has enabled me to solve, almost at sight, a further problem on which I had been spending fruitless months. He derives the hexlet with its attendant trio in general from a very simple assemblage, by inversion, namely, from a central sphere surrounded by a ring of six equal spheres between two parallel planes, or spheres of zero bend. The latter invert into two of the spheres of the trio, the central sphere being the third.

To the engineering type of mind at any rate, if not the mathematical, it would be a help to the better understanding of the hexlet if there were some fixed point that could be regarded as the centre and which could be taken as the origin of co-ordinates. I have known for long that the points of serial contact of the infinite necklace of serially touching circles, all in contact with a pair of circles which touch each other, lie on a circle the bend of which is half the difference of the bends of the pair. I therefore expected that the six serial contacts of the hexlet would lie on the surface of a sphere, the centre of which would form the natural fixed point desired.

It is obvious from Dr. Morley's originating assemblage that this is true. The plane midway between the two parallel planes, which contains the six serial contacts and the six contacts of the individual beads of the hexlet with *one* of the trio, also inverts into a sphere the surface of which cuts all the twelve contacts enumerated orthogonally and passes between the other two members of the trio at their point of contact tangentially. Its radius and centre must therefore be those of the 'circle of contacts' of the plane figure, when the assemblage is sectioned through the plane containing the centres of the trio. That is, the centre of this 'sphere of contacts' lies on the line through the centres of the two of the trio between which it passes tangentially, and its bend is half the difference between the bends of these two. I have established this *ab initio*.

The beauty of these propositions, concerning four mutually touching circles and five mutually touching spheres which underlie these assemblages, is that there is absolutely no distinction mathematically between the four circles or five spheres, respectively. It follows that there are, centred in the plane containing the centres of the trio, three different 'spheres of contacts' to each hexlet, intersecting each other, all of which contain the six points of serial contact of the hexlet, and each of which in addition contains the six points of contact of the individual beads of the hexlet with one of the trio, and the point of contact of the other two. This obscured the problem until Dr. Morley's letter furnished the clue. I had actually obtained the correct solution, but since only two of the trio were involved had rejected it as impossible !

Since the line of intersection of spheres is a plane circle, and the six serial contacts of the hexlet lie on the intersection of three spheres, the latter must intersect in the same circle, so that their centres must be in line. I am indebted to Mr. Hodgkinson here for identifying this line for me as one of the axes of similitude of the trio (see Casey's "Sequel to Euclid", 2nd ed., p. 84). It follows that the centres of the six spheres of the hexlet and the six serial contacts lie in a plane. This much simplifies the nature of the assemblage.

The bend, $\varkappa$, of the circle of six serial contacts of the hexlet is given by

$$3\varkappa^2 = \alpha^2 + \beta^2 + \gamma^2 - (\alpha\beta + \alpha\gamma + \beta\gamma)$$

or

$$3\varkappa^2 = \alpha^2 + \beta^2 + \gamma^2 - \rho^2,$$

where α, β, γ and ρ are the bends of the trio and of the circle inscribed in the triangle joining their centres, respectively. The second form is applicable without ambiguity only when all the bends are positive. By similar reasoning, the contacts of the six beads of the hexlet with each of the trio, respectively, must also lie on a plane circle.

Any hexlet may be fixed and the individual bends of the trio varied, so long as both the sum, and the sum of the squares, of the three bends remain unchanged. So long as the bends of any hexlet associated with a given trio are all positive, one of the bends of the trio may be made zero without changing the hexlet. Thus every such hexlet may be touched by two planes on either side of and equally inclined to the plane of its centres (since it is symmetrical with respect to this plane). This interesting property was discovered experimentally for some of the hexlets of the model illustrated in NATURE (Jan. 9, 1937, p. 78) by Mr. F. March, the mechanic of the Old Chemistry Department, who constructed it. In the limiting case where the bend of one of the beads of the hexlet at its minimum becomes zero, the two planes converge into one normal to the central plane of the hexlet.

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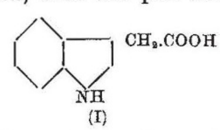
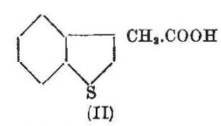
Synthetic Plant Growth Hormones

INDOLE-β-acetic acid or heteroauxine (I) is a product found in normal urine and is well known to have great growth-promoting action on plants, as shown by both the bending of the decapitated oat (*Avena*) and the pea curvature tests.



(I)



(II)

Thionaphthene-β-acetic acid (II) has been synthesized as follows : Thionaphthene → β-bromothionaphthene → thionaphthene-β-carboxylic acid chloride → β-thionaphthene acetic acid, using the reaction of F. Arndt and B. Eisert[1]. The compound melts at 109°, and has a much smaller growth activity than might be expected from its similarity with (I). The oat and pea tests are given by concentrations greater than 1 in 70,000 and 1 in 100,000 respectively, whilst (I) gives a response in dilutions about thirty times greater. It is interesting that an isomeric thionaphthene acetic acid, m.p. 141°, in which the position of the acetic acid group is not yet known, has about the same activity towards peas as (II), but is without effect on oats. A positive response by one method of testing and a lack of response by another method is not unknown ; K. V. Thimann[2] shows that coumaryl-*l*-acetic acid gives a positive pea test but no activity in the oat bending test. This and other evidence indicates that the two tests are not necessarily strictly comparable.

Naphthalene-α-acetic acid is several times more powerful than (II), as shown by both oat bending