A Simple Relation between the Quantities e, c and h

IF e is the negative electron charge, which reckoned in C.G.S. is $4\cdot7755 \times 10^{-10}$ of an electrostatic unit, and if c is the velocity of light *in vacuo* ($2\cdot9986 \times 10^{10}$ cm./sec.) and h is Planck's Constant of Action which is $6\cdot55 \times 10^{-27}$ erg-seconds, then, as is well known, the quantity $hc/2\pi e^2$ is a pure numeric generally given as equal to 137, or perhaps, 137·3.

If we absorb the 2π we can express the relation of e, c and h as follows: The quantity $e^2 = 22\cdot805 \times 10^{-20}$ is of the dimensions of energy multiplied by length or erg-centimetres. The quantity h (which I venture to name 1 acton) is of the dimensions of energy multiplied by time or erg-seconds. Hence the quantity e^2/h is of the dimensions of a velocity and is equal to $22\cdot805 \times 10^7 \div 6\cdot55 = 34\cdot82 \times 10^6$ cm./sec. Also the velocity of light, c, is $29{,}986 \times 10^6$ cm./sec. and $29{,}986 \div 34\cdot82 = 861$. But the number 861 is the product of three prime numbers, namely, 3, 7 and 41, the first, third and twelfth primes—omitting to count 1 and 2 as primes.

Also 861 is a triangular number of the form $n(n+1)/2$ or $(41 \times 42)/2$, and in addition it is the sum of the natural integers $1 + 2 + 3 +$ etc. up to 41. A triangular number denotes the number of equidistant dots which can be arranged in an equilateral triangle, namely, 3, 6, 10, 15, 21, etc. Also approximately $\pi = 3\frac{1}{7}$.

Hence we can write the equation

$$\frac{h \times c}{e^2} = 3 \times 7 \times 41 = 861;$$
$$= 3 \times 7 \times 37 + 3 \times 7 \times (3 + 1);$$
$$= (3\tfrac{1}{7} + 3\tfrac{1}{7})\ 137.$$

The numbers 1, 3 and 7 are trinal numbers of the form $n^2 + n + 1$, where n is given values of 0, 1 or 2. The velocity represented by e^2/h is nearly 1/861 of that of light *in vacuo*.

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Electrodynamics of Macroscopic Fields in Supraconductors

THE conception of a 'pure supraconducting state' has proved to be useful for the description of those phenomena of supraconductivity in which no phase transitions into the non-supraconducting state are implicated. In the pure supraconducting state the supraconductor shows no macroscopic electromagnetic field ($E = 0$, $B = 0$). It has been shown[1] that this state can be described by a consistent theory in which the idea of an infinite conductivity is replaced by the conception that, by a general relation, the supracurrents are connected with the magnetic field. The latter penetrates into the supraconductor only as deep as the supracurrents flow.

On the basis of this theory of the pure supraconducting state, a magnetostatics of the so-called 'intermediate state' has recently been developed[2] of which the characteristic feature is the appearance of a macroscopic magnetic induction B. In the intermediate state, the supraconductor is imagined as consisting of many separated microscopic pure supraconducting elementary regions, in an analogous way to the elementary regions of spontaneous magnetisation in a ferromagnet. The free energy density f of such an intermediate state has been calculated as a function of the magnetic induction, and it has been found, for $|B| \leqslant H_T$, where H_T is the magnetic threshold value, that

$$f = -\tfrac{1}{2}H_T^2 + H_T\,|B| \quad . \quad . \quad . \quad (1)$$

A generalisation of this theory is wanted for the case where an *electric* field is also present. While the general case still presents some difficulties, a very elementary conception can be developed for the special case, in which:

(a) the direction of the electric field E is everywhere perpendicular to the magnetic induction B and parallel to the macroscopic current J;

(b) the actual surface energy of the supraconducting state appreciably exceeds the lower limit derived from thermodynamics[3].

Possibly the result that follows has a more general validity than can be ascertained at present.

From (b) one can infer that the dimensions of the supraconducting regions into which the supraconductor as a whole splits up are big compared with the depth to which the magnetic field can penetrate into these regions, and that therefore all macroscopic fields are chiefly due to the normally conducting inclusions. Then to a first approximation we are justified in neglecting all 'penetration' effects and may simply set the microscopic magnetic field equal to 0 in the pure supraconducting regions (whereas the microscopic electric field vanishes there exactly). The normally conducting regions may be assumed to have the form of gaps with nearly equidistant boundaries; they will, according to (a), contain the electric field orientated perpendicular to the boundaries and the magnetic field parallel to them.

Let e and h be the microscopic field strength in such a gap, and let ξ be the volume of the gaps per unit volume of the supraconductor; then the macroscopic E and B are given by

$$E = \xi e \quad \text{and} \quad B = \xi h \quad . \quad . \quad . \quad (2)$$

On the boundary between the two phases the Maxwell pressure $\frac{1}{2}(h^2 - e^2)$ will be in equilibrium with the pressure $\frac{1}{2}H_T^2$ due to the free energy of transition into the supraconducting state (which amounts to $-\frac{1}{2}H_T^2$ per unit volume):

$$\tfrac{1}{2}(h^2 - e^2) = \tfrac{1}{2}H_T^2 \quad . \quad . \quad . \quad . \quad (3)$$

or introducing the macroscopic quantities (2) into (3), we obtain the following equation, which determines ξ:

$$\xi = \frac{\sqrt{B^2 - E^2}}{H_T} \quad . \quad . \quad . \quad . \quad (4)$$

The free energy f per unit volume consists of two parts:

(i) the 'electromagnetic energy' f_1 localised in the normally conducting gaps:

$$f_1 = \xi\,\tfrac{1}{2}(h^2 + e^2).$$

(ii) the 'internal free energy' f_2 of the pure supraconducting part, which comprises a fraction $1 - \xi$ of the total volume:

$$f_2 = (1 - \xi)\,.\,(-\tfrac{1}{2}H_T^2).$$