Letters to the Editor

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NOTES ON POINTS IN SOME OF THIS WEEK'S LETTERS APPEAR ON P. 1036.

CORRESPONDENTS ARE INVITED TO ATTACH SIMILAR SUMMARIES TO THEIR COMMUNICATIONS.

Theory of the Stern-Gerlach Effect

IN continuing the development of the theory of the masses of protons and electrons, according to which m_p, m_e are the two roots of the equation

$$10\,m^2 - 136\,mm_0 + m_0 = 0,$$

I have obtained a confirmation which seems of interest. Considering a hydrogen atom, and introducing a magnetic field by a gauge transformation, the theory is found to give values of the magnetic energy which agree with the results of the Stern-Gerlach experiment—results commonly (but, as it would seem, wrongly) supposed to show that the proton has 5/2 units of spin. It is difficult to describe this application apart from the rather comprehensive theory—contained in a book now in the press—to which it belongs; but the following will perhaps indicate the line of treatment.

As in celestial mechanics, we analyse the motion of the two particles forming the hydrogen atom into an external motion, represented by a mass $M = m_p + m_e$ moving with the centre of mass, and an internal motion, represented by a mass $\mu = m_p m_e/(m_p + m_e)$ associated with the relative co-ordinates. Correspondingly, we have external wave functions ψ_e, φ_e similar to those of a free electron or proton, and internal wave functions ψ_i, φ_i which are the well-known functions determining the internal quantum states of the atom. Each wave function is a superposition of elementary wave functions distinguished by parameters which we denote collectively by α; denoting the co-ordinates (x, y, z, t) collectively by x, we denote the elementary functions by $\psi(x, \alpha)$. An essential difference between the external and internal wave functions is that $\psi_e(x, \alpha)$ is a continuous function of α, but $\psi_i(x, \alpha)$ exists only for discrete values of α.

In current practice, the difference between continuous and discrete wave functions is inadequately considered (more especially in formulating the relation between double and simple wave functions); and my determination of the masses m_p, m_e was obtained by showing that, *in order to validate the current practice*, it is necessary that the masses should satisfy the equation above-mentioned, or equivalently that M/μ should be $136^2/10$. More precisely, it is the general dynamical equations which are validated by assigning these masses to the elementary particles. But, if we apply a gauge transformation, continuous and discrete wave functions must still be distinguished; for $\psi_i\varphi_i$ represents a density in a 4-dimensional volume element dx, and $\psi_e\varphi_e$ represents a density in a 10-dimensional volume-element $dxd\alpha$. (The number of dimensions of $dxd\alpha$ is found in the investigation of the masses, and supplies the denominator 10 in the above value of M/μ.)

We employ an imaginary gauge transformation to create a fictitious electromagnetic field in the same way that an acceleration of the co-ordinate frame is employed in Einstein's theory to create a fictitious gravitational field. If the 4-dimensional volume-element is multiplied by $e^{-2i\kappa_\mu x_\mu}$, the 10-dimensional element is multiplied by $e^{-5i\kappa_\mu x_\mu}$. The corresponding densities $\psi_i\varphi_i$, $\psi_e\varphi_e$ are changed in the inverse ratio; so that ψ_i, φ_i become multiplied by $e^{i\kappa_\mu x_\mu}$, and ψ_e, φ_e become multiplied by $e^{5i\kappa_\mu x_\mu/2}$. (It is necessary to mention that the wave functions ψ, φ used in the present theory undergo the same gauge transformation, φ being defined differently from the corresponding function in Dirac's theory, which would undergo the inverse transformation to ψ.) It follows that for the same transformation of gauge, and therefore for the same macroscopic electromagnetic field, the momentum operators are:

For discrete wave functions, $-i\partial/\partial x_\mu + \kappa_\mu$;
For continuous wave functions, $-i\partial/\partial x_\mu + (5/2)\,\kappa_\mu$.

The factor 5/2 is carried through into the mutual energy of the particle and field; so that the particle of mass M with continuous wave functions has a magnetic moment proportionately 5/2 times as great as the particle of mass μ with discrete wave functions. The common interpretation of the experiment confuses the fictitious particle corresponding to the external wave function with the proton, and that corresponding to the internal function with the electron, M and μ being very nearly equal to m_p and m_e, respectively.

The foregoing result applies to strong magnetic fields. A magnetic field determines a definite time-direction, namely, that with respect to which it is purely magnetic, the electrical components vanishing. When the field is strong, the internal states of the atom are coupled to this time-direction. When the field is weak this coupling ceases, and the 'time' in the internal state is a co-ordinate of a different character. Although usually denoted by t, it is a (linearised) interchange co-ordinate. The effect of a gauge transformation on this co-ordinate is rather difficult to investigate; but, so far as I can make out, it is gauge-invariant. This would make the volume-element for internal wave functions effectively 3-dimensional, and thus change the Stern-Gerlach factor from 10/4 to 10/3. There appears to be some experimental evidence which supports this expected modification in weak fields.

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