Letters to the Editor

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NOTES ON POINTS IN SOME OF THIS WEEK'S LETTERS APPEAR ON P. 187.

CORRESPONDENTS ARE INVITED TO ATTACH SIMILAR SUMMARIES TO THEIR COMMUNICATIONS.

The Fundamental Paradox of the Quantum Theory

UNDER the above title, Prof. G. Temple[1] deduces the paradox from the general principles of the quantum theory, that any two operators representing physical variables must commute. If this conclusion be correct, it would involve fatal consequences; therefore we should like to point out that the commutability of operators does not follow from the general principles, but only from the particular and rather arbitrary principle of symmetrisation, which Prof. Temple claims to have deduced, but which is a new hypothesis in reality.

It is generally accepted that "physical variables $a, b, c, \ldots$ are represented by symmetric linear operators $A, B, C, \ldots$ in Hilbertian space; and the representation satisfies the following conditions:

$$a^2 \rightarrow A^2,\ \lambda a \rightarrow \lambda A \ (\lambda \text{ being an ordinary number})$$
$$a + b \rightarrow A + B."$$

Furthermore, it is also admissible to conclude from the relation $ab = \frac{1}{2}(a+b)^2 - \frac{1}{2}(a-b)^2$, which is also a c-number relation, that $ab \rightarrow \frac{1}{2}(AB + BA)$. This way of symmetrisation is unique, if products of only two physical variables are considered. In the case, however, of using products of more than two physical variables, this symmetrisation can no longer be derived uniquely from a c-relation. For then it is possible to convert artificially the case of two variables into the case of three variables, having then several possibilities of symmetrisation.

We put, for example, $A = (FF^{-1}).A$, where F is an operator and F^{-1} its reciprocal. Assigning $F^{-1}A = D$, then evidently $ab = fdb$.

According to Temple's symmetrisation

$$fd.b \rightarrow \tfrac{1}{4}(FD + DF)B + \tfrac{1}{4}B(FD + DF).$$

Here, and in the general case of three variables, we will choose, however, in a more symmetrical way the sum of all six permutations of the three variables.

All the different possibilities of symmetrisation give the same result only then, if the variables commute. Hence we cannot conclude that all variables must commute because a special prescription of symmetrisation does not exist in quantum mechanics.

Note added in proof.—Further consideration shows that, starting from the above representation of physical variables by linear operators as assumed by Prof. Temple, the correct symmetrisation of products of physical variables should be made by forming the sum of all the permutations of the variables.

A detailed report on the question of symmetrisation will be published later.

H. FRÖHLICH.
E. GUTH.

Institut für theoretische Physik,
Universität,
Wien.
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[1] G. Temple, NATURE, 135, 957, June 8, 1935.

To remove any possible misunderstanding, let me say explicitly that the argument of my previous letter was not intended as a serious demonstration that any two operators representing physical variables must commute, but as a definite proof of the existence of some serious and fundamental defect in the form of modern quantum theory. The criticisms to which the argument has been subjected have concerned either the conditions which were assumed to be satisfied by the representation of variables by operators, or the method by which the representation of a triple product was obtained.

With regard to the first group of criticisms, it is sufficient to reply that the assumed conditions are universally accepted, and that the denial of any one of them is as fatally destructive of the present form of the quantum theory as is the acceptance of the commutability of any pair of physical operators. To say that the same variable need not always be represented by the same operator renders it impossible to interpret any operational formula: to deny that $a + b \rightarrow A + B$, unless A and B commute, makes it impossible to construct any Hamiltonian operator as the sum of the operators representing the kinetic and potential energies: to deny that $a^2 \rightarrow A^2$ invalidates the whole theory of the momentum operators.

Criticisms of the second kind have usually taken the form of the assertion that the operator representing abc is

$$\tfrac{1}{24}\{(A + B + C)^3 - (-A + B + C)^3 - (A - B + C)^3 - (A + B - C)^3\},$$
$$\equiv \tfrac{1}{6}(ABC + BCA + CAB + CBA + ACB + BAC).$$

Certainly, this is *a* legitimate deduction from the premises, but it is not the *only* possible deduction. I agree with Messrs. Fröhlich and Guth that in the case of triple products a unique form of the representation cannot be derived from c-relations between variables. But I deny that I have employed a "particular and rather arbitrary principle of symmetrisation . . . which is a new hypothesis in reality". I have based my deduction on the three premises admitted by Messrs. Fröhlich and Guth, and I challenge them to point out at what stage in my argument any further assumption has been made. Any triple product abc is also a double product of a and bc, or of b and ca, or of c and ab, and its representations are therefore deducible by the recognised rule for all double products, that is, that

$$xy \rightarrow \tfrac{1}{2}(XY + YX).$$

This is the gist of my argument, and *qua* argument it is invulnerable.

What, then, is the origin of the paradox? It can only arise from the fundamental concept of the representation of the variables of classical physics by the operators of quantum theory. The assertion that such a representation exists is the form taken