The procedure just outlined leads to a purely formal mathematics, which contains formal rules of demonstration but is incapable of making significant logical inferences. In particular, that a formula which has the form of a contradiction cannot arise among demonstrable formulæ is incapable of formal proof since formal mathematics does not contain such a significant proposition. That a system of axioms should not lead to contradiction is, however, an essential point, and it is here that the essence of the *Beweistheorie* lies. Hilbert therefore adjoins :

(5) Parallel to mathematics we have metamathematics, which allows significant judgments on mathematics. Such judgments must be intuitive (*anschaulich*), directly evident inferences such as $1 = 1$.

The application of this principle to the proof that a given system of axioms cannot lead to contradiction is as follows. Let P denote the property that each demonstrable formula has *not* the form

$$\mathfrak{A} \ \& \ \overline{\mathfrak{A}}.$$

It must first be proved in full finite detail that each axiom has the property P. It must then be proved that the application of the rules of inference (4) does not destroy the property P. This proof must also be so devised that it can be carried out in full finite detail in every particular case. Assuming that this can be done, it follows that the given system of axioms cannot lead to contradiction. This is, in fact, equivalent to the proof that $1 \neq 1$ cannot be the end-formula of a demonstration. For certain systems of axioms the proof is completely carried out in this volume. Hilbert is convinced that it will ultimately be possible to carry out the proof for a set of axioms on which the whole of analysis can firmly rest.

It cannot be denied that the *Beweistheorie* has serious critics. In fact the publication of the first volume was delayed, and a second volume was required, on account of certain results of Gödel which seemed to point to the impossibility of carrying out the full programme. Another serious difficulty is the meaning of words, which necessitated the recasting of the chapter on the *Aussagenkalkul* to avoid an interpretation put by Scholz on this subject in Hilbert and Ackermann's "Grundzüge der theoretischen Logik", an interpretation other than that intended by the authors.

Nevertheless, the *Beweistheorie* has cleared the ground in many fundamental problems, has produced sharp formulations of some aspects, and has suggested means of tackling the problems thus raised. Even if one were seriously to entertain the idea that Hilbert has reduced mathematics to a game, it would have to be admitted that the game is certainly not an easy one. Perhaps one quotation may be allowed.

"Die Methoden der Analysis sind in einem Ausmass erprobt, wie wohl sonst kaum eine wissenschaftliche Voraussetzung, und sie haben sich aufs glänzendste bewährt. Wenn wir diese Methoden unter dem Gesichtspunkt der Evidenz kritisieren, so entsteht für uns die Aufgabe, den Grund für ihre Anwendbarkeit aufzuspüren, so wie wir es überall in der Mathematik tun, wo ein erfolgreiches Verfahren auf Grund von Vorstellungen geübt wird, die an Evidenz zu wünschen übriglassen."

L. M. MILNE-THOMSON.

The Electronic Structure and Properties of Matter: an Introductory Study of certain Properties of Matter in the Light of Atomic Numbers. Being Vol. 1 of "A Comprehensive Treatise of Atomic and Molecular Structure". By C. H. Douglas Clark. Pp. xxvi+374. (London: Chapman and Hall, Ltd., 1934.) 21*s*. net.

THIS book, written by a chemist for chemists, sets out to review recent progress in atomic and molecular theory and to correlate various properties of matter such as cohesion in terms of the electronic structure of atoms. The first section is devoted mainly to a description of the quantum numbers of electrons in atoms, the principles underlying the electronic structure of atoms and the characteristic features of the Periodic Table. The quantum numbers used in the old orbit theory are described in some detail and then a brief account is given of the modifications found necessary in the new quantum mechanics.

This historical method of approach has its disadvantages, for it necessitates the description of symbols which are afterwards discarded. The description of modern quantum numbers is brief, and there is little indication of the interpretation which can be given to them, so that the reader will have no option but to memorise the list of symbols given. Once the quantum numbers are dealt with, the author proceeds more happily to the discussion of the electronic structure of the transition elements and the rare earths and the explanation of some of their characteristic properties. In the second section of the book, methods are discussed of determining the sizes of atoms, ions and molecules, electric and magnetic properties of the elements and the various types of cohesive forces.

The treatment is light rather than profound, and is apt to be sketchy, and to convey the impression of a series of abstracts of papers. None the less, the book provides an easy introduction to a subject which is likely to become of increasing interest to chemists and is not easily accessible elsewhere. Doubtless many will be stimulated by this broad survey to pursue the subject further, and they will be helped to do so by the excellent list of references at the end of each chapter, a feature which adds considerably to the value of the book.