Letters to the Editor

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Notes on points in some of this week's letters appear on p. 1030.

Correspondents are invited to attach similar summaries to their communications.

Mathematical Psychology of War

This is relevant to the Naval Conference. A letter in Nature of May 18 was intended to show, by reference to historical facts, that a certain pair of equations did represent, at least crudely, the behaviour of nations prior and subsequent to the Great War. In order to proceed to deductions, it is convenient now to change the notation. The hypothesis aforesaid is that

$$\frac{dx}{dt} = ky - \alpha x + g \qquad . \qquad . \qquad . \qquad (1)$$

$$\frac{dy}{dt} = lx - \beta y + h \qquad . \qquad . \qquad . \qquad (2)$$

in which x denotes the variable preparedness for war of one group of nations, y that of the opposing group ; t is the time ; k and l are 'defence coefficients' ; α and β are 'fatigue-and-expense coefficients' and g and h are measures of dissatisfaction with the results of treaties. Each of k, l, α, β, g, h is regarded as temporarily constant. Of these, k, l, α, β are each positive ; but g and h may have either sign, and will be positive for the 'have-nots', negative for the 'haves'. The international situation is thus represented by a point (x, y) in a plane. Let us think of this point as a particle moving in accordance with the equations. If the particle be tending towards plus infinity of both x and y, then war looms ahead. But if the particle be going in an opposite direction the prospect is peaceful.

If dx/dt be zero, the equation (1) represents a straight line ; similarly for dy/dt and equation (2). The intersection (x_0, y_0) of these two straight lines is the point of balance of power. It is given by

$$x_0 = (kh + \beta g)/(\alpha\beta - kl), \ y_0 = (lg + \alpha h)/(\alpha\beta - kl)$$
$$(3), (4)$$

But, unfortunately for those who live under a policy of balance of power, the point of balance has several awkward possibilities. First, by a rare chance, $\alpha\beta$ might equal kl so that the lines would be parallel and no point of balance would exist. Secondly, it is quite probable that the point of balance may be situated in one of the quadrants where x or y or both are negative ; so that balance could be attained only by negative preparedness for war, that is, by positive preparedness for co-operation. Thirdly, although the particle is undoubtedly in equilibrium when it coincides with the point of balance of power, yet that equilibrium may be unstable.

To investigate stability the equations have been solved by a standard method via the auxiliary equation

$$m^2 + (\alpha + \beta) m + \alpha\beta - kl = 0 ; \quad . \quad . \quad (5)$$

and it is found that the particle, wherever it begins, will ultimately arrive at the point of balance (x_0, y_0) if

$$kl < \alpha\beta, \qquad . \qquad . \qquad . \qquad . \qquad (6)$$

whereas the particle will ultimately go off towards infinity if

$$kl > \alpha\beta. \qquad . \qquad . \qquad . \qquad . \qquad (7)$$

That is to say : the international régime is unstable if, and only if, the product of the two defence coefficients exceeds the product of the two fatigue-and-expense coefficients.

In the unstable régime, the particle will tend towards one or other of two opposite infinities according to where it begins. We may classify the starting points by making a transformation to polar co-ordinates (r, θ) centred at the point of balance, so that

$$x - x_0 = r\cos\theta, \quad y - y_0 = r\sin\theta \qquad (8), (9)$$

For then

$$\frac{d\theta}{dt} = l\cos^2\theta + (\alpha - \beta) \cos\theta \sin\theta - k\sin^2\theta, \ . \ . \ (10)$$

whence it follows that there are always two real lines given by

$$2k \tan\theta = \alpha - \beta \pm \sqrt{(\alpha - \beta)^2 + 4lk} \ . \ . \ (11)$$

on which $d\theta/dt = 0$. The particle cannot cross these lines. They form barriers dividing the plane into four angular portions. In particular, if $\alpha = \beta$ and $l = k$ the barriers are at $\theta = \pm \pi/4 \pm \pi$. When the balance of power is unstable $(kl > \alpha\beta)$ the barrier at $\theta = -45°, +135°$, or in general at

$$2k\tan\theta = \alpha - \beta - \sqrt{(\alpha - \beta)^2 + 4lk}$$

is of profound significance, for it divides the international plane into two portions, one of a drift towards war, the other of a drift towards co-operation.

The 'constants' are liable to secular change. The progress of engineering and chemistry has notably decreased α and β, so that if l and k were unchanged, the international régime would now be much more unstable than it was a century ago. The defence coefficients k and l are influenced by the suspicion or goodwill distributed by print or broadcast.

Over all, it must be remembered that the equations describe the rivalries of two groups having populations of the same order of magnitude. In the new tentative practice of 'collective security', one population is many times greater than the other, and a quite different course of events may be expected.

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Chemical Action caused by Neutrons and Gamma Rays and the Effects of these Agents on Colloids

FOR more than a year we have been engaged in investigating the action of neutrons and γ-rays : (a) in affecting the stability of colloids, and (b) in promoting chemical reactions. As sources of γ-radiation we have used platinum containers filled with radium sulphate, or radium emanation sealed into glass tubes surrounded by a few millimetres of lead. Neutrons were obtained from sources similar to these to which metallic beryllium had been added. Since all the neutron sources used emitted γ-rays, every titration was carried out in triplicate on (1) a non-irradiated control ; (2) a specimen irradiated by γ-rays alone ; (3) a specimen irradiated by neutrons and γ-rays from the same or an equal source. The strengths of the sources ranged from 30 mC to 150 mC and the periods of irradiation from a few hours to two weeks.

Irradiations were usually carried out in the following manner. The sources were enclosed in a test tube which contained either water and a metallic lead filter, or a solution of borax and a cadmium filter. This tube was surrounded by a glass vessel containing the liquid under examination. The test tube and vessel were again surrounded by a large mass of water, or placed in a cavity in a block of paraffin wax. In this way, full advantage was taken of the now well-known methods of slowing down and absorbing neutrons.

Amongst the chemical actions investigated have been the oxidations of potassium metabisulphite and sodium bisulphite, and the decomposition of hydrogen peroxide. In each case it was found that the action provoked by a pure γ-ray source was increased by using a source of equal strength which also emitted neutrons.

The following colloids have been investigated : Hydrosols of silver and gold, the sulphides of arsenic and cadmium, and vanadium pentoxide and ferric hydroxide. When prepared in the standard way it was found that a source emitting both neutrons and γ-rays increased the stability of the negatively charged colloids and decreased the stability of the positively charged colloids. A similar but definitely smaller effect was obtained with pure γ-ray sources of the same strength. Anomalous results could, however, be obtained when foreign electrolytes were intentionally added to the colloidal sol before irradiation.

In all the actions outlined above, it was found that the effect which must be attributed to the neutrons alone was of the same order of magnitude as that due to the γ-rays alone. The greater efficiency of slow neutrons in producing these effects is strikingly exhibited, since with the sources used approximately one neutron was produced per hundred thousand γ-ray quanta emitted. So far as we are aware, no previous observations of the effect of neutrons on chemical reactions or colloidal stability have been published.

This investigation was begun in the hope of discovering a method of detecting neutrons which would be independent of the usual techniques of inducing radioactivity or the use of electrical counters. Two methods, which possess the great advantage of integrating the effects produced until they are easily measured by standard chemical methods, are indicated above.

The similarity of the effects due to neutron and γ-ray irradiation would seem to indicate that the commonly accepted views on the mode of action of the latter are in need of revision.

A fuller account of this work will be published shortly.

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Production of Cosmic Ray Showers at a Considerable Depth below Ground-Level

IN order to find whether showers are produced by the penetrating cosmic rays which reach considerable depths below ground-level, we have performed experiments in Holborn Underground station (by permission of the London Passenger Transport Board) at a depth corresponding to 60 m. of water. The vertical intensity of the radiation at this depth, as measured by the rate of occurrence of coincidences between three counters in a vertical plane, is about one-fifteenth that at ground-level. To count showers we used five counters, arranged in a pentagon formation, so that at least three particles are needed to discharge the five counters simultaneously. The occurrence of coincidences is therefore in itself definite proof of the presence of showers. The quintuple coincidence method is preferable to the triple coincidence method as usually used hitherto, as in the latter case a coincidence can be produced by two particles. The presence of showers can then be proved only by showing that there is an increase in the rate of occurrence of coincidences when a sheet of, say, lead is placed over the counters, the dimensions and disposition of the lead being such that coincidences due to particles originating in it must be due to at least three particles. The detection of showers then entails the establishment of a difference between rates with and without the lead, and these may be nearly equal.

Our preliminary results have shown that (a) showers are produced in the earth above the counting apparatus ; (b) the shower rate is increased several fold when the group of counters is surrounded by lead ; (c) the thickness of lead for maximum shower production is about 1·6 cm., that is, about the same as at ground-level ; (d) for this thickness the ratio of the rate of shower production to vertical intensity is not very different from that at ground-level under the same conditions.

These conclusions are not in agreement with those of Auger[1], who has reported the absence of showers under 8 m. of earth ; or those of Pickering[2], who has reported a rapid diminution in the ratio of the rate of shower production to intensity at depths down to 10 m. of water. The difference seems to be explicable in terms of the counter dispositions used in the various cases, and we shall discuss this point when our results are published in full.

Since there is good reason to believe that protons, owing to their large mass, should produce very few showers, we can conclude from our results that a considerable part of the radiation which penetrates 60 m. of water must consist of positive or negative electrons.

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Dec. 5.

[1] Auger et Bertein, J. Phys., 6, 253 : 1935.
[2] Pickering, Phys. Rev., 47, 423 ; 1935.