lower oxides was undertaken by Berzelius in 1816, but the industrial application of tungsten was delayed until 1847, although an alloy with iron had long been known. After the Paris Exhibition in 1900 the use of tungsten in steel manufacture became general and three years later filaments of the metal were used in electric bulbs.

Very interesting data are given relating to the production of tungsten. Before 1910, Australia was the largest producer in the world; then Burma and the United States gained the lead for a few years, but since 1918 the greater part of the world's output has come from China. In technical research Germany led the way until her supplies were cut off during the War.

Tungsten is used as a hardening component of certain alloys and also as a pure metal. Carbide of tungsten is an important constituent of the hard alloys, and tabulated lists of references to the patent specifications are grouped according to the melting or sintering properties of the alloys. Recent industrial applications of the pure metal have been largely due to its low volatility and high melting point. It has practically replaced all other metals as lamp filaments and it is used for making electrodes, thermo-elements and many other appliances. This volume is packed with details about the properties of tungsten and its compounds and the literature has been revised to April 1933.

Modern Geometry

Analytical Geometry of Three Dimensions. By Prof. D. M. Y. Sommerville. Pp. xvi+416. (Cambridge: At the University Press, 1934.) 18*s.* net.

THIS is a textbook: it is not too difficult for the average 'honours' student, it includes the material usually required for examinations; yet it also includes less usual subjects such as line-geometry, cubic and quartic curves, cubic surfaces, ruled surfaces, higher space, the Veronese surface, the application of matrices, invariants and invariant factors; above all, it conveys something of the true spirit of modern geometry. For many years geometrical research has been made both easier and more effective by projective methods and space of more than three dimensions, but this had not hitherto found recognition in any sufficiently elementary English textbook.

Though primarily algebraical and three-dimensional, the book provides a more unified knowledge of geometry by the frequent use of purely geometrical methods and by excursions into one, two, four and five dimensions. It does not deal with birational transformations or topology. Its reasoning is in general particularly concise and intelligible, though occasionally condensation renders the argument obscure. A carefully chosen first course of reading makes it really suitable for those whose knowledge of co-ordinate geometry is limited to elementary conics. References to more detailed expositions are too scanty.

The treatment of the straight line, plane and quadric accords with examination requirements, but rectangular Cartesians are systematically supplemented by homogeneous co-ordinates, line co-ordinates, matrices, and (most conspicuously and successfully) the circle at infinity and other projective methods. There are useful notes on imaginary elements. Beginners may possibly find the projective investigation, as an introduction to focal properties, somewhat overwhelming. The invariants of two quadrics are treated extensively. Segre characteristics are sufficiently explained. Certain chapters would benefit from a more comprehensive selection of heuristic examples.

Elsewhere pure projective geometry tends to predominate. In one chapter co-ordinates are established without reference to metrical considerations: difficulties are avoided by assuming, openly, the fundamental theorem.

The curve theory includes developables, the (plane) Plücker relations, Lüroth's theorem, the twisted cubic and its polar system, twisted quartics of both species, curves upon a quadric. A chapter on line geometry deals with the quadric in five dimensions. The treatment of surfaces includes curvature, polar surfaces, the effect of isolated singularities on class, ruled surfaces, double curves, the cubic surface and its twenty-seven lines, the Steiner surface, the Veronese surface, normal varieties, the cyclide.

Misprints are not serious; (y) for (x), on p. 371, is slightly confusing. A few statements require emendation: three concurrent lines on a cubic surface do not necessarily meet in a node; quartic surfaces are known containing as many as sixty-four lines; despite Continental nomenclature, the general point of a double curve should not be confused with a binode; part of 14·683 is incomprehensible. Terms are too freely used in an unconventional sense without warning; for example, linear series (for regulus), self-conjugate tetrahedron.

The excellence of this work emphasises how much geometry loses by the recent death of its author.