Auroras, Electric Echoes, Magnetic Storms

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RECENT reports by E. V. Appleton and his associates, Norwegian and other[1], have brought out the complex connexions between the optical and magnetic phenomena of the upper atmosphere. The considerations that follow touch only the fringe of this interesting subject: but it may be permitted to record them before they pass out of memory.

Assuming waves short enough to permit analysis by ray-propagation, there would be two paths of transit from one place to another not too distant, one straight across, the other by reflection from what has been appropriately termed a ceiling aloft. This latter is represented by a caustic surface, belonging to the source supposed of coherent periodicity, at which all the rays are turned back tangentially: none can get across it, unless the medium is discontinuous consisting for example of banks of reflecting ionic clouds. Some frequencies have no ceiling, or one only of limited extent. The numerical densities (N) of electrons at the apices of the ray paths, which practically lie along the caustic, are determined at once by the optical law of refraction, that $\mu \sin z$ is constant along a ray; for the directional angle z to the zenith is $\frac{1}{2}\pi$ at the apex, so that the value of μ there is $\mu_0 \sin z_0$ for the position of the observer. Unless the direction of emission is at very small angle (z_0), μ is thus a moderate fraction of the initial value μ_0, about unity, at the level of emission. Either then N at the apex is a moderate number, rather less than for vertical reflection as *infra*, or else z_0 is very small so that all the rays that reach the ceiling start off nearly vertically and bend sharply, or else the analysis by rays is not applicable to the lengths of waves concerned.

It has been found, without doubt, that rays are returned straight back along the vertical path, the highest point of the gradually sloping caustic surface being as above overhead. Its exact position can scarcely be determined: but certainly the plane for which μ vanishes—the velocity of propagation there becoming infinite or the medium optically rigid—which is readily estimated, lies beyond the caustic. The familiar formula for frequency $p/2\pi$ and electrons of mass m,

$$\mu^2 = \mu_0 - \frac{4\pi N e^2 c^2}{mp^2}, \quad \frac{p}{c} = \frac{2\pi}{\lambda}, \quad \frac{e}{m} = \frac{7}{4}\cdot 10^7, \quad e = \frac{3}{2}\cdot 10^{-20},$$

gives, for N per cubic cm. and λ in cm., $\mu^2 = 1 - \frac{4}{5}10^7 N\lambda^2$. Thus for waves of the order of 300 metres, μ^2 would vanish when N has risen to the moderate value $\frac{1}{7}10^5$, and for 30 metres to $\frac{1}{7}10^7$. The intensity of ionisation which thus arrests propagation by waves is small, perhaps much beyond expectation. This does not, however, mean that a cloud of ions, of diameter more than a few wave-lengths, would collapse by any essential instability. Vanishing index means that the medium is electrically absolutely rigid for these lengths of waves, so that disturbances of such length could not get into it at all, would be turned back or in part smothered: infinite index, as *infra*, would mean that it is flaccid, with like results: this now holding good whether treatment by rays is applicable or not, thus in this rough estimate avoiding the complexities of gyro-radiational analysis, as exemplified in S. Goldstein's work[2]. If radiation of various wave-lengths is sent up from the source, each constituent is turned back before the stratum for which N has the limiting value (inversely as λ^2) is reached. This is the foundation on which is based the exploration of atmospheric strata by vertical radiation, as initiated and extensively carried out by Appleton and his colleagues, uncertainties regarding oblique reflections thus not entering into the estimates.

Carrying the analysis further: in a magnetic field H_0, in the simplest case for the two cyclic waves ($+H_0$ and $-H_0$) travelling directly along H_0, the expression for μ^2 involves in the denominator $mp^2 + eH_0p$ instead of mp^2. One of the two waves becomes obliterated by μ^2 trending to infinite value, so that the velocity fades to nothing at a stratum where p or $2\pi c/\lambda$ has fallen to $\frac{7}{4}\,10^7\,H_0$, which for values of H_0 of the order of the earth's magnetic field would be when λ is more than half a kilometre. The radiation which gets through is then solely the conjugate cyclic component. Here N does not occur at all in this estimate: but that could not mean that even a very sparse distribution of electrons would prohibit one set of component waves if the impressed field H_0 (or rather $H_0\lambda$) were great enough. It means that there could not be cyclic radiation of this kind with what few electrons there may be present playing a part in it; this is because in circular orbits such as they would have to follow the centrifugal reaction mp^2r could not adapt itself to compensate the electrodynamic force eH_0r, and therefore such participating orbits could not subsist: but when the number of ions is small cyclic radiation can travel in the ordinary manner, only slightly disturbed by their irregular motions.

Specially close connexion of magnetic storms and the Aurora Polaris with anomalies in wireless radiation is reported by Appleton. This contrasts, of course, with the extremely subordinate influence of magnetic fields on the short waves of physical optics, except for ferromagnetic metals. A conceivable presumption would be that the aurora is due to lasting local pulsations on a large scale, so of long period, excited somehow by a local cause large enough and of abrupt type, produced conceivably by arrest high up of an ionised torrent from outside sufficiently concentrated to require relief by propagation in waves: the incidence of such long undulations on the molecules of the lower rarified atmosphere could produce the light of the banded auroral curtains. As these bands lie along the direction of the magnetic field this would imply facility of spiralling transmission along that direction. Probably also it has been already explored whether the auroral light shows traces of circular polarity.

Long ago the ascription of terrestrial magnetic changes to electric currents circulating in the upper atmosphere was in favour[3], until the recognition that all currents are made up of convections of ions disturbed that view by the high electron densities implied. Yet there seems no help for it if atmospheric ionic views are to be persevered with: thus in recent careful discussion, S. Chapman[4] has not been deterred from densities even up to 10^{10} electrons per cubic centimetre in positions very far up in the abnormal atmosphere. The superior mobility of the negative electrons is there the dominating influence, for compensating positive must be present.

An arresting feature of Prof. Appleton's pairs of fragmentary graphs, giving heights of reflection in terms of frequency, as directly observed, such as may be on his view connected with the two cyclic components into which the radiation is split by the earth's magnetic field, is that though of irregular form, they show rather close repetition of features, differing mainly by a shift along the axis of frequency. This suggests search for an analytical correspondence between them, which it is not hard to pursue for the simplest illustrative case of radiation along the direction of the magnetic field H_0. The co-ordinates of the graphs are p, giving the frequency $p/2\pi$ of the waves, and the altitude z of the reflecting layer estimated by the rough criterion of vanishing index μ. More generally, N being some assigned function of z, the graph may relate to any constant value of μ or c/c'. The equation of propagation (cf. my "Math. and Phys. Papers", vol. 2, p. 651, as alone here accessible) is, if φ denotes the single complex electric variable $P + \iota Q$ and d/dt is ιp,

$$-\frac{d^2\varphi}{dz^2} = Kc^{-2}p^2\varphi + \frac{4\pi cNp^2}{mp^2 - 4\pi cH_0p}\varphi$$

This type of differential equation is familiar for other modes of waves, and has been tractable for some special forms of N as expressed in terms of z. When, however, N changes not too rapidly with z, a simple harmonic type $e^{\iota pt}e^{-\iota nz}$, so that $d^2\varphi/dz^2$ is $-(\mu/c)^2\varphi$, is a first approximation and, φ dividing out, gives μ^2 in terms of p and N. Transition is made to the conjugate wave train by change of sign of H_0. To explore correspondence of the types indicated, we restrict to the case of μ nearly vanishing, when

$$N = \frac{Km}{4\pi ec^2}\left\{(p + p_0)^2 - p_0^2\right\},$$

where p_0 is $2\pi cH_0/m$, being half the critical frequency above described. Only in the circumstances of short waves is p_0^2 small compared with p^2 and roughly can be neglected: then N is determined by $p + p_0$, and the graph for (N,p) is merely displaced opposite ways from a central position by adding $+p_0$ or $-p_0$ to the abscissa p according to the sign H_0. But the actual graphs belong to long waves.

In the next simplest case, when the magnetic field H_0 is transverse to the waves (*loc. cit.*, vol. 2, p. 655, where some misprints are to be set right), the displacement of the graphs now involving $(p_0/p)^2$, would be large for a magnetic long wave. (The condition for μ to vanish then takes a simple form $Lp = \pm N$.) The actual case, that of field oblique, along the line of magnetic dip, would be nearer the former: though intricate, it may possibly be worth the trouble of exploring, if that has not already been done by Goldstein (*loc. cit.*). Down to the higher atmospheric density of the auroral levels the incident electron streams could scarcely penetrate.

As Appleton originally suggested, and is confirmed by Ratcliffe's records, the upper reflection is probably due to an independent layer, and both reflections can be split by the magnetic field in a manner to which this analysis applies.

By good fortune, however, the magnetic complications here described appear largely to compensate themselves. A cyclic wave-train of dextral chirality going up would be returned as a train of the same chirality, in absence of a magnetic field: imposing the earth's magnetic field would affect them in opposite ways: thus so far, if there is no error here, the magnetic delays in ascent and return should cancel, provided they are along the same path, so that there would be no delay on this ground in reception between the two cyclic components of a wave-train: the actual delay would arise from their reflection being at different levels, and would afford a measure of the interval, and thence of the difference in electron density. The conjugate cyclic polarisations, each *received* reversed, would remain as a test whether the splitting of the beam is of magnetic type.

A reflecting layer would have to be fully established in a fraction of a wave-length, thus rather abruptly for short waves. Thus it would have to fade away more gradually above to avoid thin-plate phenomena: this and the much larger density N that must be attained notwithstanding for short waves may provide clues for exploration.

The unexpected result, that whether a magnetic field be present or not, a quite small density of ions entirely upsets the optical elasticity of space as regards long waves, provides a cause preserving ionised gaseous clouds of astronomical size, for example in the interiors of stars, from rapid dissipation or dispersal in bulk; in fact saves an ionised region from lapse to uniformity by internal radiation.

The description[5] of a recording apparatus developed by J. A. Ratcliffe and E. L. C. White has just attracted my notice. The automatic photographic records, of which examples are printed, convey a sense of actuality to which mere verbal description could scarcely attain. The continuous run shows the intervals of time during which double reflections are present, and the heights, sometimes very great, from which they appear to arrive. We may thus know in time as much about the earth's upper atmosphere as we seem to know about that of the sun. The abrupt changes in vertical atmospheric structure recorded in crossing sunset and sunrise meridians agree closely with the early suspected cause of the related disturbances in long-distance signalling.

As regards the rather uncertain concept of group-velocity (cf. *loc. cit.*, vol. 2, p. 546), so familiar for a long time in hydrodynamics, it is an affair, as Hamilton first described it, of an *unlimited* train of regular waves with humps of increased amplitude at intervals, which travel through the waves with a velocity of their own, dp/dn as against p/n for the basic waves. These humps do not constitute a train of waves of the same type, for their average amplitude would be null, though they could be receivable as in wireless practice by different apparatus. It may not be too wild to imagine a permanent train of waves sent out and encountering a dispersive region in which the group velocity approaches zero owing to very sharp curve of dispersion, so that these humps remain nearly stationary, but when the train on which they ride is terminated, they relieve themselves by propagation forward and backward: but in any case these are scarcely the circumstances of the alleged echo of long delay.

V. M. Slipher reports regular occurrence of flashes of auroral spectrum at sunrise and sunset. One can conceive an upper stratum ionised by the solar radiation, and a lower by the long electric waves that can disturb electric reception.

These long atmospheric waves would be indicated by disturbed electric reception, but they would not affect the magnets; that would be due to the vast exciting torrents of solar electrons flashing past the earth and partly arrested aloft. They would require a compensating fall into the sun (perhaps spiralling in the sun's rather strong magnetic field) and, as it would be at slower velocity, there would be a solar electric charge. Yet, as I understand, G. E. Hale could not find any trace of Stark effect of resulting solar electric field : which would be adequately explained if the electric charge, being of course a surface sheet, lies outside the layer which emits the light. The penetration of the cosmic radiation, if it carry a charge, or part of it, into the earth has been in like manner assigned as a cause maintaining the earth's electric charge[6]: by the usual estimate it would have to replenish the static charge of the earth every ten minutes. The spatial density of compensating charge falling back into the sun would be considerable if it fell slowly : but there is scarcely ground for connecting the fixed spectral lines of some double stars with an atmosphere of that kind. This principle of emission in one mode and compensating absorption in another, is far-reaching : thus it is the foundation of the Einstein theory of radiation by projected and absorbed 'photons'.

The characteristic feature of the modern spectral theory, expressing itself in sporadic transitions between energy levels, is that, while it aims at inclusion of the Hamiltonian dynamical analysis, each line has its own configuration of the source, without any overtones such as were a necessary part of a vibrational theory. The equation of Hamilton-Jacobi and the related one of Schrödinger would belong not to an atom but to a cosmos, thus coming into line towards the various universal modes of statistics. In asymptotic limit[7] (λ large) the two schemes, Hamilton's (generalised) rays and Schrödinger's potential, come into agreement.

[1] NATURE, **132**, 340, Sept. 2, 1933.
[2] *Proc. Roy. Soc.*, 1928.
[3] cf. *Phil. Mag.*, Jan. 1884 ; "Math. and Phys. Papers," vol. 1, p. 28.
[4] *Terrestrial Magnetism*, 1931-33.
[5] *Proc. Phys. Soc.*, Feb. When this was written I had not seen their records for *short waves*, and their cyclicity in the magnetic field, which is not very far from vertical, in *Phil. Mag.* for July. Clearly there is much to be learned here.
[6] cf. Kolhörster, H., NATURE, **132**, 407, Sept. 9, 1933.
[7] cf. Dirac's "Quantum Mechanics," p. 121 ; G. D. Birkhoff, *Proc. Nat. Acad.*, March 1933, p. 339 : also Levi-Civitá, *Bull. Amer. Math. Soc.*, Aug. 1933. An early attempt toward such correlation is in the writer's "Papers", vol. 2 (1928), p. 809.

Anniversary of the Asiatic Society of Bengal

ON January 15 the Asiatic Society of Bengal celebrated the 150th anniversary of its foundation by an afternoon conversazione in the Indian Museum, and a banquet in the evening followed by a special anniversary meeting. The conversazione was attended by the Mayor of Calcutta and about five hundred of the leading citizens of Calcutta. It took the form of a garden party on the lawn of the Museum and special and interesting collections of exhibits, consisting of paintings lent by the Academy of Fine Arts, copies of old documents from the Imperial Records Department, mostly of the eighteenth century and some concerning the Asiatic Society, paintings of plants from the Botanical Survey, Javanese and Siamese sculptures lent by Dr. S. K. Chatterji, chemical and physical demonstrations arranged by the University College of Science and Technology, prehistoric and tenth century finds from the Archæological Survey, fossils, crystals and economic products from the Geological Survey, birds of Bengal lent by Dr. S. C. Law, demonstrations of the prevention and treatment of disease arranged by the School of Tropical Medicine and Hygiene, medals and coins by His Majesty's Mint, Kaffir attire, fish, Crustacea and insects from the Zoological Survey.

The banquet was honoured by the presence of His Excellency Sir John Anderson, Governor of Bengal, who is the patron of the Society, and took place in its one hundred and twenty-six year old hall, surrounded by portraits and busts of former members who have made history in Bengal. Ninety-three members and guests were present, including the consular representatives of France, Germany, Holland, Sweden and the United States of America, the Archbishop of Calcutta, the Hon. Sir M. N. Roy Chowdhury, Sir David Ezra, the Hon. Nawab K. G. M. Faroqui, Sir C. C. Ghose, the Hon. Sir A. K. Ghuznavi, Lord Sinha, the Hon. Sir B. P. Singh Roy, and Sir Jadu Nath Sircar. The toast of the guests was proposed by the president of the Society, Dr. L. L. Fermor, to which M. J. Delacour, of the National Museum of Paris replied, and also proposed the Asiatic Society, but speeches were brief, in view of the meeting afterwards.

At the special anniversary meeting, His Excellency the Governor took the chair and the president delivered his anniversary address, outlining the history of the Society, and naming the distinguished contributors to its publications, more especially in the last half century. He pointed out that many of the specialist departments and institutions founded in India originated from the Asiatic Society, in particular the Indian Science Congress, and mentioned the proposals which had been made for the formation of an Indian Academy of Sciences to affect co-ordination between these various interests in the sphere of science.

Following the president's address, congratulatory messages were read from His Excellency the Viceroy, the Mayor of Calcutta, the League of Nations, Prof. C. Rockwell Lanman, Sir George Grierson and Sir Thomas Holland (honorary fellows). Seven addresses were read, from the British Museum, the Linnean Society, the Zoological Society of London, the Batavian Society of Arts and Sciences, the Indian Institute, Oxford, the Schopenhauer Society, Frankfort, and the Prussian Academy of Sciences Congratulations were presented by 26 delegates from 58 learned institutions, and in all 19 countries were represented—Australia, Austria, Belgium, Ceylon, Canada, France, Federated Malay States, Germany, Great Britain, Hungary, Italy, Japan, Netherlands, Spain, Sweden, Switzerland, Tasmania, United States and India.

Twelve honorary anniversary members were elected—six in letters and six in science : Prof. Arthur Christensen of Denmark, Prof. Taha Husain of Cairo, Sir John Marshall, lately Director-General of Archæology in India, H.R.H. Prince Damrong Rajahubhab of Siam, Dr. Rabindranath Tagore, Dr. J. Van Kan, law member of the Council of the Viceroy of the Dutch East Indies, Sir Sidney Burrard, lately Surveyor-General in India, Prof. Albert Einstein, Sir Sven Hedin, Prof. Alfred Lacroix, Dr. Henry Fairfield Osborn and Lord Rutherford.

In his speech, His Excellency the Governor stressed the vigour of the Society in spite of its age, its permanence since the days of the French Revolu-