in accordance with the picture given by Weiss, Heisenberg and others. In still stronger fields a small component I_p, will also occur, which will vanish again at saturation. Kaya's result for his crystal 8, for which the calculations are rather simple, are, on the whole interval of field strengths, in satisfactory agreement with Gans's constants of anisotropy[2], if a term (3) is introduced into the expression of the free energy. Abrupt changes in the slope of the magnetisation curve, as reported also by Kaya, can, of course, scarcely be explained on the present theoretical basis.

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* In accordance with the general use, we employ the term 'free energy', though the term 'thermodynamic potential' for the same function would be more appropriate.

[1] S. Kaya, *Z. Phys.*, **84**, 705; 1933.
[2] R Gans, *Phys. Z.*, **33**, 924; 1932. E. Czerlinsky, *Ann. Phys.*, **13**, 80; 1932.

The Neutron in Quantum Mechanics

IT can be shown with the help of Dirac's relativity equation that the neutron can be properly placed in the scheme of the wave-mechanical theory. Dirac's Hamiltonian for a hydrogen-like atom in polar coordinates is

$$H/c = -e/c.A_0 - \varepsilon p_r - i\varepsilon\rho_3 jh'/r - \rho_3 mc$$

in which ε and ρ_3 are to be taken as the matrices

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

respectively, leading to the equations

$$\left(\frac{1}{a_1} + \frac{\alpha}{r}\right)f - \left(\frac{\delta}{\delta r} - \frac{1}{a} + \frac{j}{r}\right)g = 0$$

$$\left(-\frac{1}{a_2} + \frac{\alpha}{r}\right)g + \left(\frac{\delta}{\delta r} - \frac{1}{a} - \frac{j}{r}\right)f = 0$$

where the wave-functions are $\psi_1 = e^{-r/a}f$, $\psi_2 = e^{-r/a}g$, $a_1 = \frac{h'}{mc + H'/c}$, $a_2 = \frac{h'}{mc - H'/c}$, $h' = h/2\pi$, $a = (a_1 a_2)^{1/2}$ and $\alpha = e^2/h'c$ is small[1].

f and g must be finite series, if a is real, or $H' < mc^2$. The radius of the smallest orbit will be the smallest value of r that makes $(\psi_1\bar{\psi}_1 + \psi_2\bar{\psi}_2)$ a maximum, since Dirac's wave-function is r times the Schrödinger function χ.[2]

The Schrödinger function χ is supposed to be uniform and finite everywhere, but as its physical significance is bound up with the probability $\chi\bar{\chi}dV$ of the electron being found in the elementary volume dV, it may be permissible to relax the conditions a little. A priori, there does not seem to be any reasonable ground for disallowing some form of singularity at the origin where there is a centre of force. We may tentatively suppose that χ must be such that $\int\chi\bar{\chi}dV$ should exist and be finite for any region finite or infinite. In the present case, a constant value for f and g would indicate that the electron falls into the centre. Leaving this case out, we assume the solutions

$$f = c_0 r^{1/2}, \quad g = c_0' r^{1/2},$$

which satisfy the wave equations above if $\alpha^2 = j^2 - \frac{1}{4}$. α being small ($7{\cdot}3 \times 10^{-3}$), j may be taken as $\pm\frac{1}{2}$ approximately. For the closest approach to the centre, we take $j = -\frac{1}{2}$ and the two solutions of the wave equations are

$$\psi_1 = -e^{-r/a} r^{1/2}, \quad \psi_2 = ke^{-r/a} r^{1/2} \text{ where } k = (a_2/a_1)^{1/2}.$$

On normalising, the smallest distance will be given by the value of r which makes $re^{-2r/a}$ a maximum, that is, $r = \frac{1}{2}a$.

Since the series for f and g have been taken as a single term, the quantum number n is zero and the case is really an exceptional one to which Dirac's calculation of the eigenvalues of H does not apply. In fact no eigenvalues of H can be determined.

Since $a = \frac{h'}{(m^2c^2 - H'^2/c^2)^{1/2}}$, taking H'/c as being not nearly equal to mc, we get $r = 1{\cdot}9 \times 10^{-11}$ cm. as the lowest permissible value, which is about 280 times smaller than the radius of the smallest Bohr orbit. This value of r gives a consistent value for H'/c when substituted in the expression for the Hamiltonian, as it makes the term h'/r of the same order as mc, and the difference between H'/c and mc of the same order as either of them.

It will also be clear that Dirac's restriction of j to integral values is based on the uniformity of the wave-function, a condition which has been relaxed above. It appears that half odd integral values of j are, in fact, permissible.[3]

In conclusion, even if the value of r does not agree with the value for the radius of the neutron (10^{-12} cm.) suggested at present, it may be of interest as the lower limit of the distance permissible under the Coulomb force.

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[1] "Principles of Quantum Mechanics", § 78, p. 253.
[2] *ibid.*, § 45, p. 143.
[3] See, for example, Born and Jordan, "Elementare Quanten Mechanik", § 27, p. 142.

$K\alpha_1\alpha_2$ Doublet of Phosphorus

SOME time ago I published the results of an investigation on the effect of chemical constitution on the $K\alpha_1\alpha_2$ doublet of the elements sulphur[1] and chlorine[2]. With sugar as analysing crystal I have now made an introductory, analogous investigation on the $K\alpha_1\alpha_2$ doublet of phosphorus. The emitting substances were violet phosphorus, sodium hypophosphite and sodium phosphate.

The experimental arrangements were the same as those for my investigation of chlorine. The phosphorus lines, excited by X-rays, were measured relative to the $K\beta_1$ line of calcium, excited by electron impact. The phosphorus lines were photographed in the third order, the reference line in the sixth one.

Measurement of the results shows a difference of the wave-length amounting to 0·8 X.U. between the doublet of the hypophosphite and that of the phosphate. Relative to the doublet of the free element I have found the following displacements towards shorter wave-lengths:

for the hypophosphite doublet	1·7 X.U.
for the phosphate doublet	2·5 X.U.

A displacement towards shorter wave-lengths of the doublet of a phosphate was earlier found by Bäcklin[3], who excited the doublet by electron impact.

A closer investigation on the doublet is in preparation.

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Lund. Aug. 26.

[1] *Z. Phys.*, 77, 778; 1932.
[2] *Z. Phys.*, **83**, 85; 1933.
[3] *Z. Phys.*, **33**, 547; 1925.