Modified Field Equations with a Finite Radius of the Electron

THE attempts to combine Maxwell's equations with the quantum theory (Pauli, Heisenberg, Dirac) have not succeeded. One can see that the failure does not lie on the side of the quantum theory, but on the side of the field equations, which do not account for the existence of a radius of the electron (or its finite energy = mass).

I have developed a new method of the quantisation of the electromagnetic field in such a way that the four independent variables (time-space) are treated absolutely symmetrically and the principle of relativity is trivially fulfilled. From very general principles of quantum theory (superposition of states, linearity of the equations for the amplitudes of probability) one can deduce how Maxwell's equations have to be modified. In the classical theory, they are equivalent to the statement that the Lagrangian is given by $L=\frac{1}{2}(H^2-E^2)$; in the new theory, this expression is replaced by a linear function of the field components, the coefficients of which are non-commutative quantities of a type similar to those in Dirac's theory of the electron. In the limit, where the classical theory should hold, the new Lagrangian does not go over into the above given expression, but into

$$L=\frac{1}{a^2}\sqrt{1+a^2(H^2-E^2)}$$

where a is a constant of the dimensions r_0^2/e (e = charge, r_0 = radius of the electron), and only in the limit $a \to 0$ does this tend to

$$\frac{1}{a^2}+\frac{1}{2}(H^2-E^2).$$

The simplest case of a central-symmetrical field independent of time ($H=0$, $E=-\text{grad}\ \varphi(r)$) gives for φ the differential equation

$$\frac{d}{dr}\left(\frac{r^2\dfrac{d\varphi}{dr}}{\sqrt{1-a^2\left(\dfrac{d\varphi}{dr}\right)^2}}\right)=0,$$

with the solution

$$\varphi=e/r_0 \cdot \int_{r/r_0}^{\infty} \frac{d\xi}{\sqrt{1+\xi^4}},$$

where e is a constant of integration and r_0 is a length chosen so that $a=r_0^2/e$. For $r \gg r_0$, one has Coulomb's law, $\varphi \to e/r$, but for small values of r/r_0 the new potential is finite and tends to $1 \cdot 85\ e/r_0$.

The form of the general expression for L ensures the relativistic invariance. Thus there is no difficulty in calculating the properties of a moving electron on the basis of the classical theory. But the importance of the new Lagrangian L seems to lie in the possibility of a systematic quantisation of the field.

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The Uncertainty Principle

THE familiar application of this principle is to a particle, such as an electron, and from it we learn that errors, Δq in the positional co-ordinate and Δp in the momentum, must occur such that:

$$\Delta q\ \Delta p \sim h.$$

This principle is bound up with the ideas in the wave theory of matter and is accepted as part of that theory, in which it is embedded. It is interesting to note, especially in view of a possible generalisation of the principle to systems more complex than a particle, that a little boldness of statement might have led substantially to it in the earlier days of the quantum theory. The work of Planck and Sommerfeld led to the recognition of a cell-like structure in phase space. Thus on a diagram representing the co-ordinate, q, along one axis and the momentum, p, on the other, the whole area was divided into small rectangles $\Delta q\ \Delta p$ of area h. One might have postulated that elementary areas of smaller size in this diagram had no physical significance or that points within the area could not be distinguished from one another. The statement of this postulate would thus be:

$$\Delta q\ \Delta p \nless h,$$

which is only another form of the uncertainty relation.

This is the method of approach adopted in the discovery of the principle of minimum proper time and it would have been difficult to accept it without the background of the wave theory.

The interest here is that the generalisation of the idea gives a principle of indeterminacy for systems of n degrees of freedom, namely:

$$\Delta q_1 \ldots \Delta q_n\ \Delta p_1 \ldots \Delta p_n \nless h^n,$$

for the phase space now becomes composed of cells of 'volume' h^n.

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Curious Atmospheric Refraction Effects

WHILE in a rowing-boat on the ocean during a sultry afternoon this summer I observed a very peculiar series of refraction phenomena that seem worth describing. A thunderstorm was brewing in the north-east far enough away so that the thunder was just audible. There was no sunshine. The wind had dropped and the sea was gently undulating. A small steamer was passing westward some three miles distant. When I first saw her she was almost hull down (apparently). In a few minutes she loomed up so that the forecastle was high, although the midship section was normal. A little later the cabins disappeared entirely and all one could see was an unbroken line of black hull, the funnel and the masts. Next she again appeared hull down, only to emerge with complete hull and upperworks. Then once more she seemed to sink so that her deck was awash. These changes took place as one gazed at the boat and were most startling. The hull seemed slightly longer at the time when only the deck showed but this may have been imagination on the part of the observer.

I attribute the effects to irregular currents of cool air from the approaching storm gently flowing over the water. A number of years ago I witnessed a similar case, in which a receding steamer seemed to sink to her upper deck at three miles, to emerge safely within the next mile. I wonder if anyone else has noticed such freak refraction as the first case cited?

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