Milne's Theory of the Expansion of the Universe

By DR. G. C. McVITTIE

PROF. E. A. MILNE has recently put forward in these columns[1] and also elsewhere[2] a theory designed to account for the phenomenon of the recession of the spiral nebulæ on purely kinematic grounds. The basic idea of this theory is both simple and elegant and can be described as follows.

Consider a swarm of particles moving with constant, but otherwise arbitrary, velocities in an infinite Euclidean space. The swarm has spherical symmetry around a point at which we can suppose an observer A to be stationed who employs a rectangular set of axes (x, y, z) and a Newtonian time t to describe the motions of the particles. Suppose that at $t=0$ the swarm occupies the interior of a sphere S around A, who is at $x=0$, $y=0$, $z=0$, the space outside S being empty. Then as time goes on, the particles will necessarily move out of S. For example, those near the surface of S having velocities directed away from A will immediately move out of S whilst others may have to traverse a portion, or the whole, of the sphere before they get outside it. But it is obvious that, given a sufficient lapse of time, the particles will sort themselves out so that eventually only the slowest moving ones will remain inside S. All the particles will, moreover, be now receding from A. If the original distribution of the velocities was continuous and included the velocity zero, there will always be some particles left near to A, and in fact it is not difficult to see that the average velocity of the particles at a distance r from A at time t is r/t, so that there is an approximate correlation of velocity with distance. Thus the behaviour of the swarm will be very similar to that of the spiral nebulæ observed in Nature, practically all of which are moving away from us with velocities which increase linearly with the distance.

The above result still holds when the distribution of particles is supposed to be infinite, provided only that there is an initial concentration of particles in the neighbourhood of A.

Milne now asks the question: What must be the space and velocity distribution of such a swarm so that a second observer, A', moving relatively to A with constant velocity and using a Euclidean space-frame (x', y', z') and a Newtonian time t', should have an equivalent view of the swarm? The principle of special relativity will ensure that the views of A and A' are consistent: to make them identical, Milne puts forward an extended principle of relativity, namely, "not only the laws of nature, but also the events occurring in nature must appear the same to all observers, wherever they be, provided their space-frames and time-scales are similarly oriented with respect to the events which are the subject of observation". These two principles are sufficient to enable him to determine, purely as a problem in statistics, the space and velocity distribution function of the swarm. It is, in terms of A's co-ordinates,

$$\psi\left(\frac{Z^2}{XY}\right)\frac{dx\,dy\,dz\,du\,dv\,dw}{c^6 Y^{5/2} X^{3/2}} \tag{1}$$

where

$$X=t^2-(x^2+y^2+z^2)/c^2,\quad Y=1-(u^2+v^2+w^2)/c^2,$$
$$Z=t-(ux+vy+wz)/c^2.$$

This is the number of particles counted by an observer who sees the swarm centred around himself, in the volume-element x to $x+dx$, y to $y+dy$, z to $z+dz$, having velocities lying between u and $u+du$, v and $v+dv$, w and $w+dw$. All observers, of course, are supposed to be at the origin of their space-frames. The second observer A' will have an identical picture of the swarm if, and only if, his co-ordinate system is related to that of A by a Lorentz transformation of type

$$t'=\beta(t-Vx/c^2),\quad x'=\beta(x-Vt),\quad y'=y,\quad z'=z;$$
$$\beta=(1-V^2/c^2)^{-\frac{1}{2}}. \tag{2}$$

A' then sees the swarm centred around himself, and both A and A' count the same number of particles for equal volume-elements and equal velocity-intervals in their respective co-ordinate systems. Observers of this kind we call of 'class A'.

The condition that $x=0$, $y=0$, $z=0$, $t=0$ for A corresponds to $x'=0$, $y'=0$, $z'=0$, $t'=0$ for A' is necessary for the invariance of (1) under the transformation (2) and hence is necessary for the equivalence of the two pictures of the swarm. Consider now a third observer B who is *defined* as being at rest with respect to (say) A. He uses a co-ordinate system of type

$$t''=t,\quad x''=x+a,\quad y''=y,\quad z''=z, \tag{3}$$

where a is a constant giving the distance from A to B as measured by either of them. Milne recognises that B will not have an equivalent view of the swarm and attempts to overcome this difficulty by identifying B with an observer A'' of class A who is moving past B with velocity a/t at the instant t, having been at A at time $t=0$. But it is not easy to see how this can be justified either on mathematical or on physical grounds. It would thus appear that Milne's extended principle of relativity is equivalent to a definition of a certain class of observers, namely, those who have equivalent views of the swarm of particles. These observers all coincided with one another and with the centre of the swarm 'simultaneously' and all set their clocks to read $t=0$ when this happened. This moment can also be supposed to correspond to the moment of greatest concentration of the swarm and is so taken by Milne.

The distribution function

$$\frac{(\text{constant})\;dx\,dy\,dz\,du\,dv\,dw}{Y\,[c^2\sum(x-ut)^2-\sum\{v(z-wt)-w(y-vt)\}^2]^{3/2}} \tag{4}$$

has been specially studied by Milne: it arises from a particular solution* of his equation for ψ and corresponds to the case of rectilinear motions of the particles of the swarm as viewed by an observer of class A. This is therefore the case when the interactions of the particles are neglected. It follows that at $t = 0$ there were an infinite number of particles at the origin. At any subsequent $t > 0$, particles which were initially at the origin and moving with velocity V, will have got to $r = Vt$ and evidently these particles again have an infinity in their distribution at that distance. Thus these infinities recede from the observer with velocities proportional to their distances from him. The general effect of gravitation is to remove these infinities and to replace them by density-maxima in the distribution of the particles. Thus finally we have a set of groups of the original particles which appear to any observer of class A to recede from him according to the law $r = Vt$. However, there still remain singularities in the distribution of the groups on the sphere $r = ct$, so that the latter are strongly concentrated there. Milne identifies the groups with the spiral nebulæ, thus obtaining the law of velocity of recession proportional to distance. The concentration towards $r = ct$ is shown to be consistent with the apparent uniform distribution of the nebulæ owing to the rapid diminution of brightness due to the high velocity an object acquires on approaching this bounding sphere.

From the observed correlation of 500 km. per sec. for every 3·25 million light years distance of a nebula and the law $r = Vt$, it follows that $t = 2 \times 10^9$ years have elapsed since the system of the nebulæ was at its greatest concentration.

We may in conclusion contrast Milne's theory with that based on general relativity. The latter deals only with the group of observers at rest with respect to one another, who are observing a set of particles also at rest with respect to the observers in the sense that the total momentum of the particles is assumed to be zero. The particles initially are in a state of unstable equilibrium and the expansion is due to what may be called a 'cosmical repulsion'. In Milne's theory the particles are initially in motion and are not in an equilibrium state. The observers considered are not at rest with respect to each other. The expansion is simply due to the fact that the particles in a certain region have higher velocities on the whole than those elsewhere. Both theories are in accordance with observation and it seems impossible to decide definitely for or against either so long as the phenomenon of the recession of the nebulæ, in isolation from all other phenomena, is to be the only criterion.

* It appears as the *only* solution in the paper quoted in (2). Prof Milne has since discovered others.

[1] NATURE, **130**, 9–10, July 2, and 507–508, Oct. 1, 1932.
[2] *Z. Astrophysik*, **6**, 1–96, Jan., 1933. See also E. Freundlich, *Naturwiss.*, **21**, 54–59, Jan. 27, 1933.

Nitrogen-Uptake of Plants

PROGRESS, due to the abandonment of traditional beliefs, is occurring to-day in agriculture as well as in other branches of applied science. At one time it was generally held that ammoniacal nitrogen had first to be converted into nitric nitrogen before it could be taken up by plants, but we now know of many plants that take up ammoniacal nitrogen directly; and at the present time doubt is being cast on a doctrine which has persisted since the days of Liebig and Boussingault, namely, that organic nitrogen has first to be 'mineralised' before it becomes available as food for plants. The work done by Prof. A. I. Virtanen and his collaborators at Helsingfors during the past few years has provided important evidence that at least some plants directly and readily assimilate organic nitrogenous compounds, and a useful summary of this work was given recently by Prof. Virtanen in lectures delivered to the Netherlands Agricultural Society at Wageningen and to the Chemical Society of Zurich.

Earlier experiments showed that certain legumes, for example, red clover, did not grow so well when the nitrogenous food derived from the root nodules was replaced entirely by ammonium nitrate, but that peas utilised both forms of nitrogen equally well, whilst white clover responded better to ammonium nitrate than to the food provided by the nodule bacteria. These and other observations led to the hypothesis that legumes take up their nitrogen from the nodules in the form of organic compounds, and that these compounds constitute the best form of nitrogen-nutrition for certain legumes. It was also found that these compounds diffuse into the surrounding soil, from which they are taken up by growing plants, like *Gramineæ*. Both pot and field experiments have shown that, when the pH of the soil does not fall below 6, one pea plant can provide sufficient nitrogen for two oat plants, but if the ratio of oats to peas is greater than two to one, both plants suffer from lack of nourishment.

Investigations were then made using sterilised sand and pea seeds inoculated with nodule bacteria, and it was found that the nitrogenous matter that diffused into the sand consisted entirely of organic compounds, which were almost equal in amount to the nitrogen compounds directly taken up from the nodules. It was also established that nitrogen compounds diffuse into the soil from young and fresh root nodules of the alder tree, thereby enabling the latter to assist the growth of other trees.

Analysis of the substances that diffused into sterile quartz sand from the roots of pea plants that had been strongly inoculated showed the chief constituent to be amino-acids: in percentages of total nitrogen, amino-nitrogen 77·4, ammonia-nitrogen 0, amido-nitrogen 3·30, nitrogen in volatile bases 2·73, humin-nitrogen 2·05.