General Transformation Theory in Hilbertian Space

ONE can write the Dirac's wave equation

$$\{\alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3 + \alpha_4 mc\} \psi = \lambda \psi \quad (1)$$

in the matrix representation

$$\sum_{\sigma, \kappa} \{(\alpha_1)_{\sigma\sigma}(p_1)_{\kappa\kappa} + (\alpha_2)_{\sigma\sigma}(p_2)_{\kappa\kappa} + (\alpha_3)_{\sigma\sigma}(p_3)_{\kappa\kappa} + (\alpha_4)_{\sigma\sigma} mc\} \psi_{\sigma\kappa} = \lambda \psi_{\sigma\kappa} \quad (2)$$

which will be invariant under any unitary transformation of the Hilbertian spaces of spins and momenta ($(p_i)_{\kappa\kappa}$ denote the components of the momentum matrix in the Hilbertian space).

Usually one considers only the Cartesian systems in Hilbertian space, but from a general point of view one is led to investigate the invariance of quantum laws under more general affine transformations. To assure this invariance one has to introduce the 'metric matrices' $\gamma^{\sigma\sigma}$ and $g^{\kappa\kappa}$ for the two Hilbertian spaces as new dynamical variables, and to write equation (2) as follows

$$\{(\alpha_1)_{\sigma\sigma} \gamma^{\sigma\sigma} (p_1)_{\kappa\kappa} g^{\kappa\kappa} + (\alpha_2)_{\sigma\sigma} \gamma^{\sigma\sigma} (p_2)_{\kappa\kappa} g^{\kappa\kappa} + (\alpha_3)_{\sigma\sigma} \gamma^{\sigma\sigma} (p_3)_{\kappa\kappa} g^{\kappa\kappa} + (\alpha_4)_{\sigma\sigma} \gamma^{\sigma\sigma} mc\} \psi_{\sigma\kappa} = \lambda \psi_{\sigma\kappa} \quad (3)$$

(where the covariant and the contravariant characters of the indices as usual are distinguished by their position).

To account for an external electromagnetic field, one usually introduces in the Dirac equation the potentials of the field, which causes a corresponding change in the eigenvalue spectrum of the equation. But if one starts from the general equation (3), one can always by an appropriate choice of the 'metric matrices' $\gamma^{\sigma\sigma}$ and $g^{\kappa\kappa}$ ensure that this equation gets a given eigenvalue spectrum. This follows from the well known fact that by a simultaneous non-unitary transformation any two quadratic forms $H_{\kappa\kappa}$ and $G_{\kappa\kappa}$ may be transformed into diagonal forms, one of them becoming the unit form. We can thus formulate the principle of general transformation as follows:

It is impossible to distinguish between a set of states described by a Dirac equation for a free particle in a non-unitarian Hilbertian space and a corresponding set of states, described by the usual Dirac equation in a given external field.

Thus the electromagnetic field appears to be an expression of the non-unitary metric in the Hilbertian space in the same way that the gravitational field is an expression of the non-Euclidian metric of the space-time world.

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Room Comfort

IN a note in NATURE of Nov. 12, p. 743, reference was made to the kata-thermometer as being considered too small for its purpose as an indicator of comfortable conditions in a room, and to the larger electrically controlled instrument, the eupatheo cope, described by Mr. A. F. Dufton, the surface of which is automatically kept at 75° C., a temperaturescorresponding to that of the clothed surface of the body in ordinary room conditions. This instrument was considered more suitable owing to its size.

The kata-thermometer does not indicate the rate of cooling of the body but of the bulb of the thermometer, which approximately is at the temperature of the skin of the face. The range of cooling powers

which are best for comfort were fixed empirically in conditions which were admittedly fresh and pleasant. Like the skin of the face, the kata-thermometer is very sensitive to the cooling effect of wind.

I have shown that the infra-red rays from dull red and dark sources of heat produce uncomfortable sensations in many people. They tend reflexly to congest the nose and make it stuffy, the congestion lessening the nasal air-way. These effects are set aside by cool air, for example, by a fan blowing on the face. As the kata-thermometer is very sensitive to cool warming air, it is an excellent indicator of those conditions which give a fresh feeling. The mere securing of a certain cooling power and temperature of the clothed surface is not sufficient to give comfort, and as the small size of the kata-thermometer is actually an advantage, its utility is not surpassed by that of the eupatheoscope.

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Dec. 10.

Colour-blind Motorists and Red Danger Signs

IT is not, I think, realised what a menace the colour-blind have become under present conditions; their contribution to the toll of the roads, for example, is never even mentioned. Most colour-blind persons are blissfully unaware of their abnormality and in Great Britain there are probably one hundred thousand colour-blind persons licensed to drive motor vehicles. Would-be drivers of railway trains are tested as a matter of course and some four per cent are rejected for defects in colour-vision, but no tests are imposed upon those who use the roads.

The peril of the roads is enhanced by our use of red as the sign of danger—the one colour which the ordinary colour-blind person cannot see at all—and it is instructive to don a pair of blue spectacles (copper glass) and to learn how the world appears to a daltonian. The danger is greatest, of course, at night, when the red rear-lights of vehicles, road obstruction lamps, reflectors on bicycles and beacons at cross-roads are well-nigh invisible and fail to convey any warning. Automatic light-signals which show red as a stop-sign may be a source of danger by day as well as by night.

As colour-blindness is largely congenital, it will doubtless soon be practicable to eliminate it. In the meantime all bus drivers should be tested (on November 5 one did not see my brilliant red rear-light and charged into the tail of my car), and coroners might well inquire into the colour-vision of those who at night drive down inoffensive cyclists.

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Systems of Four Immiscible Liquid Layers

THE mixture described by E. Lester Smith¹ gives a system which separates into four layers, as stated, but does not appear to be stable. A specimen made up as described at first separated into four layers, but after long standing these become two layers. One sample of the system made up in January 1931 has now formed two layers, coloured brown, and separates into these two layers on standing after agitation. The phase rule deals with systems in true equilibrium.

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¹ NATURE, 127, 91, Jan. 17, 1931.