Quantitative Estimates of Sensory Events

By Dr. ALLAN FERGUSON

AN immense mass of literature has accumulated around the problem which was stated by Fechner more than eighty years ago as that of measuring the increase of a mental intensity in terms of the relative increase of the corresponding physical energy. His problem, in fact, was that of finding a quantitative measure of certain subjective sensations.

Other qualities apart, we can say of two sensation-brightnesses that they are *equal* in magnitude, that one is *greater* than the other, that one is *just perceptibly greater* than the other; the same may be said of two sensation-loudnesses. It is an unfortunate fact that such terms as loudness or brightness are used indiscriminately, sometimes in reference to the magnitude of the sensation, sometimes in reference to the magnitude of the stimulus which produces that sensation. It is desirable to adopt a uniform practice and, in what follows, these terms will be applied solely to the subjective sensations involved.

It seems, then, that just as in the region of physical stimuli we can use such terms as *greater*, *less*, and *equal*, so in the region of sensory events we can, and do, use the same terminology. Loudness and brightness are magnitudes, in the sense that colour and shape are not magnitudes. Are they quantitative magnitudes? Can we say by *how much* one loudness, or one brightness, is greater than another? This is a fundamental problem for psycho-physics, and it is essentially the question which Fechner essayed to answer.

Obviously the answer, logically treated, demands some knowledge of what we mean by measurement, and it is perhaps unfortunate for progress in psycho-physical work that the issue has, almost from the outset, been clouded by wranglings concerning the possibility of such measurements—arguments that have neither dealt adequately with the theory of measurement nor recognised the fact that quantitative physical magnitudes are not to be classed all under one head. Happily, the human mind is, in certain matters, reasonably illogical and, in the determination of purely physical magnitudes, physicists have been content to measure first and to theorise afterwards. Overmuch theorising concerning fundamentals is apt to have a paralysing effect; and it is well to be mindful of the fate of the centipede, who

> ". . . was happy quite,
> Until the frog, for fun,
> Asked her which leg went after which,
> Which raised her doubts to such pitch,
> She lay distracted in a ditch,
> Not knowing how to run".

The psycho-physical measurements with which we are concerned have felt something of this paralysis; it is therefore a favourable augury for progress when we find Dr. R. A. Houstoun and Dr. L. F. Richardson, from different lines of approach, making quantitative measurements of sensation, without troubling overmuch to discuss their possibility.

It is perhaps simplest to approach the matter from the historical point of view. Gustav Theodor Fechner (1801–1887), physicist, poet, philosopher, and mystic, had unique qualities for the tasks which he faced—qualities, too, which were prone to raise suspicion in minds more prosaic than his own. This may account for some of the difficulties which his work encountered, as well as for the measure of success with which it met. It is essential, in order fully to appreciate those difficulties, to see Fechner against the background of his time and of his own philosophy, an animistic synthesis which should enable him "to follow consciousness itself into an underworld—a world which must then be no other than the abiding place of a general consciousness of which life is but a ripple on the surface". Fechner, early in his endeavour to establish the relations which we have mentioned, chanced upon certain observations made by E. H. Weber about 1831. Weber had noted that, in a test on weight discrimination carried out according to certain specified rules, a skilled observer who could just discriminate between weights of 29 and 30 drams, could also just discriminate between weights of 29 and 30 ounces. Further, this relative sensitiveness remained the same over a fairly wide range. As it is usually stated, if, when the magnitude of the stimulus (*Reiss*) is R the just perceptible increase is δR, then, over a certain range,

$$\delta R/R = k.$$

The value of k in this particular instance is about 1/30.

With due precautions concerning experimental technique, and correct interpretation of the psychological conditions, the law holds for the sensory qualities associated with such diverse stimuli as sound, light, pressure, and even the stimulus to the sense of smell.

It must not be imagined that the law holds over more than a limited range. It obviously fails at the threshold of sensation and, if $\delta R/R$ be plotted as ordinate against R as abscissa, a curve results which is approximately horizontal over a very limited portion only—and it is to this portion that Weber's law applies.

Whether, however, $\delta R/R$ is constant, or is some complicated function of R, is a matter of little weight for the development of the rest of the argument. Let us, for the moment, confine ourselves to the region within which $\delta R/R$ is approximately constant. It is customary at this stage to point out (and the present writer confesses to what seems to be a lapse) that Weber's law, as thus stated, is quite unexceptionable, inasmuch as it is concerned exclusively with physical stimuli—weights which may be measured in dynes or in grams weight, sound intensities which may be measured in watts per square centimetre, and so on. This is, per-

haps, not quite correct, for, after all, δR is the *just perceptible* increase—the increase corresponding to an increase δS in sensation. What we have written as δR should therefore be written as a differential coefficient, and the statement of Weber's law should read

$$\delta R/\delta S = kR.$$

Stated in this form, the law may still be regarded as unexceptionable, though it is conceivable that some purist may criticise this form on the ground that it prejudges matters by assuming the existence of an *element* of sensation. However this may be, the next step, that of an integration of the equation just propounded, has been criticised on all hands. Integrated over the region for which Weber's law is valid, we find

$$S - S_0 = k \log (R/R_0),$$

where S_0 and R_0 refer to the values of S and R at an arbitrary origin within the prescribed region. Some curious algebraical exercises are to be found in the textbooks which give alternative methods of arriving at this relation, but it must be understood that *any* process by which one steps from the original statement of Weber's law to a statement of some functional relation between S and R involves an integration, implied or overt, and consequently involves the assumption that $\Sigma\delta S = S$.

And why not? It would take too long here to discuss in detail the objections advanced, some of them apparently irrelevant, most of them variations on one theme.

Thus, William James remarks that when we consider sensations only, we are "quite unable to read any clear meaning into the notion that they are masses of units combined. To introspection, our feeling of pink is surely not a portion of our feeling of scarlet; nor does the light of an electric arc seem to contain that of a tallow candle in itself. . . . Introspection shows, moreover, that in most sensations a new *kind* of feeling invariably accompanies our judgment of an increased impression; and this is a fact which Fechner's formula disregards." Again, Stumpf says: "An sich ist und bleibt unlängbar, dass eine Empfindung nicht das Mehrfache einer anderen sein oder wenigstens nicht als solches erkannt werden kann. Mussten wir doch sonst die eine von der anderen subtrahiren, und die Rest für sich empfinden können. Jede Empfindung präsentirt sich uns als ein Unteilbares." This oft-quoted criticism has been regarded as final. None the less it possesses an inherent weakness which is best exposed by quoting Lewis Richardson's happily inspired parody—"One mountain cannot be twice as high as another. If it could, we ought to be able to subtract the one from the other and to climb up the remainder by itself. Every mountain presents itself as an indivisible lump."

Despite these criticisms, we do regularly make quantitative laboratory experiments in which, for example, it is found possible to arrange a series of grey shades from very light to very dark in what appear to be equal steps, and it is also an experimental fact that a comparison of these shades by photometric methods shows that their objective luminosities are in a geometrical progression; this, and similar observations, if interpreted in the sense that they justify the Fechner equation

$$S - S_0 = k \log (R/R_0),$$

are apparently in direct contradiction to the criticisms just quoted. How can we escape the dilemma? Historically, the escape was made by introducing the idea of sense intervals, a notion originally due to Delbœuf, and followed out by later writers. The idea is very clearly formulated by Titchener, who remarks, "the physical . . . magnitude is not a single term but rather a distance between terms. . . . We are apt to say, carelessly, that we have measured 'the highest point' of Mt. Vesuvius, when we have in reality measured, in terms of our arbitrary unit, the distance between its lowest and highest points. It is not the point that is the magnitude but it is the distance between points. So with sensations; we are apt to think of a brightness, or of a tone of given intensity, as a sensation magnitude, as itself measurable. Now the stimulus is measurable; we can measure, in terms of some unit, the amplitude of vibration of the ether or air waves. . . . But the sensation, the brightness or the tone, is just a single point upon the sense scale—no more measurable in itself than the 'highest point' on Mt. Vesuvius. The only thing that we can measure is the distance between two sensations or sense points."

This brings us to another criticism—that whenever we measure an ordinary physical quantity, we express it in terms of a unit of the same kind, of which it is a multiple or sub-multiple. Where is the unit of subjective brightness or loudness, ask the critics? Well, we can arrange a series of steps of just noticeable differences, or we can, as we have seen, in the matter of shades of grey, arrange a series of steps of equal-seeming finite differences. It seems to satisfy most writers if these are taken as unit steps for the measurement of sense intervals, rather than as units for the measurement of sensation magnitudes. But it is again doubtful whether this attitude does more than evade the main point.

Most writers on the psychological side base their criticisms on some such fundamental assertion as that quoted earlier from Stumpf. This type of argument may be paralleled from many other parts of the literature. It has one inherent weakness—it does not discriminate clearly between two types of physical magnitude.

Such magnitudes as length, mass, and volume may be very readily conceived as being built up of units spatially juxtaposed in such a way that a unit length plus a unit length gives a length of two units, and so forth. But there are other physical magnitudes of which this is certainly not true, and yet on which quantitative measurements are regularly made. What, for example, is the result of adding, in the sense just used, a density of one to a density of one? Or a unit temperature to a unit temperature?

There is a host of such quantities—magnitudes such as surface tension, viscosity, density, diffusion

coefficient, and the like, which are not fundamental magnitudes of the type previously discussed, but may none the less be measured quantitatively. Whether the magnitudes concerned are fundamental or not, a mass, a temperature (not a hotness), a viscosity, even though they may be conceived artificially as built up by the multiplication of a certain unit quantity, do surely present themselves to introspection with the same singularity as does, say, a sensation-brightness.

Moreover (we are now in the region of physical stimuli), even though a physical magnitude X may present itself to introspection as a whole, if we find that X varies with temperature in such a way that $dX/d\theta = f(\theta)$, we have no hesitation in integrating to obtain a functional relation between X and θ, and are led thereby into no contradictions. So, while fully recognising that in one instance we are making deductions from introspection concerning a physical magnitude, and in the other instance from our judgment concerning a subjective sensation, we do not find ourselves led into any morass of contradiction if we regard a just noticeable difference or an equal-appearing interval as a unit of sensation, and artificially regard any sensation magnitude as so many times greater than that unit.

This is, apparently, what Dr. Houstoun does—and in doing so ingeniously avoids difficulties arising from departures from Weber's Law—when he plots $R \div (\delta R/\delta S)$ as ordinate against (log R) as abscissa. Since $d(\log R)$ is equal to $\delta R/R$, it follows that the area included between two ordinates separated by the small interval $d(\log R)$, the x-axis, and the element of the curve is equal to δS. Consequently, if $\Sigma\delta S = S$, the magnitude of the sensation corresponding to any stimulus R, is given by the area underneath the curve up to the ordinate at the point considered. This expresses an important advance. Incidentally, Dr. Houstoun finds that the curve obtained is very closely a Gaussian probability curve.

Other minor criticisms may be briefly noted. It has been remarked that the quantity $S - S_0$ does not represent a difference between two experiences, but an experience of difference. "The expression $S - S_0$ represents a single state of consciousness, the experience of a difference. It admits neither of dissection nor of mathematical treatment." Such a statement is merely dogmatic. Again, criticism of the choice of a just noticeable difference as a unit is contained in the remark that Fechner "regarded a sensation as the sum of a number of just appreciable unit increments of sensation. . . . He maintained that the change of sensation, obtained by adding one ounce to a weight of twenty-nine ounces was absolutely the same as that obtained by adding one dram to a weight of twenty-nine drams. *Of course, were this so, an ounce and a dram should produce an equal sensation.*" The italics are the present writer's. Comment is surely unnecessary.

Dr. L. F. Richardson has approached the matter from a different viewpoint. He endeavours to measure a sensation S "by directly estimating the *ratio* of unequal intervals both much larger than the least perceptible". Thus he has, from 316 observers, obtained estimates of the redness of certain pinks, these estimates being made by putting points on a line divided into 100 equal parts, white being zero, and red 100. The inquiry has been elaborated by mixing on a colour-wheel white and red in different proportions and estimating in the same way the redness of the resulting colour. If X is the position of the mark on the red scale and the angular amount of red (measured as a percentage of 360°) is θ, R. S. Maxwell finds that the results of 35 observers may be represented by $(\theta - 156)(X + 56) = -8736$.

One word concerning the *decibel*. This may, or may not, be used to define a unit of sensation-loudness. The physical measure of the intensity of a musical note of a given pitch rises, apparently, at a much more rapid rate than does the judgment of its sensation-loudness. If we measure physical intensities in, say, micro-watts per square centimetre, we obtain a series of numbers 1, 10, 100, 1000 . . . ., or 10^0, 10^1, 10^2, 10^3 . . . . As a matter of mere convenience it may be advisable to represent these intensities by the series 0, 1, 2, 3 . . . ., and we thus obtain a logarithmic scale of intensities of which the unit has been called the *bel*. One-tenth of this unit is the decibel, and it does happen that this unit corresponds fairly closely to the just noticeable difference of sensation-loudness between two notes of the same pitch at moderate intensities. But primarily the decibel represents a unit of intensity on a logarithmic scale, and need have no more to do with sensation measurements than has a scale of cents in the realm of music.

It is evident that, despite the amount of adverse criticism that has been brought to bear on Fechner's interpretation of Weber's Law, the matter is by no means closed, and that even the long-standing evasion of the difficulties in terms of sensation-intervals may stand in need of revision. The joint discussion between Sections A and J at the York meeting of the British Association should do much to clear up the major points at issue.

Obituary

MR. H. G. WATKINS

THE tragic death on Aug. 20 of Henry George Watkins at the age of twenty-five years has removed the most promising and, indeed, the most prominent figure amongst British arctic explorers, a figure as yet too recent to be familiar to those outside a small circle.

The stages of Watkins' rapid advance to the forefront are simply told. At the age of nineteen, while still an undergraduate at Trinity College, Cambridge, he led a summer expedition to Edge Island in the Spitsbergen group. At the age of twenty he spent an arduous year in Labrador with one companion, J. M. Scott, the full story of which