Obituary.

MAJOR P. A. MACMAHON, F.R.S.

ON Christmas Day of last year, Major Percy Alexander MacMahon died at Bournemouth. By his death, the science of pure mathematics has lost a distinguished devotee with a striking individuality. Not that MacMahon was solely a scientific investigator. He had been a soldier; he retained the title of his military rank to the end of his days. He had been engaged in teaching for not a few years; his powers of exposition were marked by a clear directness that could be envied. He had been a Civil Servant; for fourteen years, until his retirement in 1920 under the age-limit, he was Deputy Warden of the Standards of the Board of Trade. But throughout all the stages in his varied avocations, MacMahon achieved and maintained a high reputation as a pure mathematician.

The outward facts of MacMahon's life may be recorded briefly. Born at Malta on Sept. 26, 1854, he was the second son of Brigadier-General P. W. MacMahon. His school was Cheltenham: on leaving school, he went to the Royal Military Academy at Woolwich and entered the Royal Artillery in 1872. He was professionally connected with that arm for many years until he was of standing for promotion to colonelcy; but he abstained from qualifying for promotion, and he retired with the rank of major. Some early years of military duty were spent in India. On his return to England he was drafted into the educational side of military training. He was appointed an instructor in mathematics at Woolwich in 1882: there, he was a colleague of his former teacher, Prof. A. G. (afterwards Sir George) Greenhill, for whose powers he ever retained the highest respect. Later, in 1890, he was appointed professor of physics in the Ordnance College; and from 1904 to 1920 he was attached to the Board of Trade, in the office already mentioned.

Concurrently with all these successive professional occupations, MacMahon was diligently engaged in research. In no long time he had established a mathematical reputation by his investigations, published in the *Quarterly Journal of Mathematics*, the *Proceedings of the London Mathematical Society*, the *American Journal of Mathematics*, and the *Proceedings of the Royal Society*. Soon, a gradually growing share in the official activities of learned societies was assigned to him. He was elected a fellow of the Royal Society in 1890: was president of the London Mathematical Society in 1894–96: was president of the Royal Astronomical Society in 1917: and for many years, either at intervals, or for long continuous periods, he was a member of the respective councils of those bodies. He also acted as a General Secretary of the British Association for twelve years: in 1914 he was appointed a Trustee: while in 1901, at the meeting in Glasgow, he was president of Section A (Mathematical and Physical Science) of that body, delivering an interesting address upon the general aspects of the subjects of his own mathematical preference.

The value of MacMahon's original work was widely recognised by the conferment of honours, such as academic and scientific corporations alone can worthily confer. Unconnected with any university by training, he received a number of honorary degrees; he was made doctor of science by Dublin in 1897 and by Cambridge in 1904, and doctor of laws by Aberdeen and by St. Andrews in 1911. It is no secret that he was all but appointed Savilian professor of geometry in the University of Oxford in succession to Sylvester, on the latter's death in 1897. In his later years, until his health broke down in 1928, when he removed to the south coast of England, he and his wife settled in Cambridge. After his honorary doctorate, and by express invitation, he had joined St. John's College, Cambridge, a college including in its foundation many personal friends such as the present Master (Sir Robert Scott), Sir Joseph Larmor, and Prof. H. F. Baker, among the mathematicians.

Nor were scientific honours less profuse than those of an academic quality. MacMahon was elected an honorary member of the Royal Irish Academy and of the Cambridge Philosophical Society. The Royal Society appointed him its representative as a governor and fellow of Winchester; and awarded him a Royal Medal in 1900 and the Sylvester Medal in 1919. The London Mathematical Society awarded him the De Morgan Medal in 1923.

Thus MacMahon's military career had gradually merged into avocations connected specially with pure science; and the worth of his scientific life had met with ample recognition. But the real crown of his scientific life is constituted by the additions to knowledge which he achieved in the course of his mathematical investigations.

MacMahon's contributions to mathematical science are contained in many separate memoirs, more than one hundred in number, and in one treatise on the grand scale, his "Combinatory Analysis" in two volumes, published (1915, 1916) by the Cambridge University Press. The development of his genius, when once he had settled into his main region of original research (and he settled early), was swift and clear, as exhibited by the sequence of topics in his memoirs regarded chronologically; and that development was maintained with a continuity which was remarkable and persistent. His earliest papers were rather scattered in their subjects: he dealt with isolated topics, with some properties of special curves or the integrations of some differential equations connected with elliptic functions. But very soon his true line had been found: thereafter, progress was steady and unhalting.

His real beginning was made not later than 1883 by a simple discovery which opened up a new field of investigation and completely transformed one range of the theory of invariantive forms, created and amplified by Cayley, Sylvester, and Hermite. In that theory it had long been known that any seminvariant—that is, an invariant or the leading coefficient of a covariant—of a binary quantic of quite general rank satisfies a central linear partial differential equation of the first order, which depends solely upon the parametric coefficients of the quantic. It was also a matter of established knowledge that, subject to one proviso, a symmetric function of the roots of an algebraic equation of quite general degree satisfies another linear partial differential equation of the first order, which likewise depends solely upon the parametric coefficients in the equation. The limiting proviso is that the partition of the symmetric function must be non-unitary: that is to say, if the symmetric function of the roots $\alpha, \beta, \gamma, \ldots$ of the equation be denoted by the expression $\Sigma\alpha^p\beta^q\gamma^r \ldots$, no unit integer shall occur in the partition $p, q, r, \ldots$ of the number $p+q+r+\ldots$ which is the weight of the function.

MacMahon's discovery (hailed as 'very remarkable' by Cayley) was that, by mere arithmetical changes connecting the coefficients of the quantic $(a_0, a_1, a_2, \ldots)(x, 1)^n$ with the coefficients of the equation $c_0x^n + c_1x^{n-1} + c_2x^{n-2} + \ldots = 0$, the critical differential equation satisfied by the seminvariants of the quantic becomes identical with the critical differential equation satisfied by the non-unitary-partition symmetric functions of the roots of the equation. These arithmetical changes, by which the coalescence is effected, are

$$a_0 = c_0,\ a_1 = 1!c_1,\ a_2 = 2!c_2,\ a_3 = 3!c_3, \ldots$$

The property, thus discovered, entails the consequence that seminvariants of the quantic and non-unitary-partition symmetric functions of the roots of the equation are formally equivalent, if the foregoing relations hold between the two sets of coefficients. For example,

$$a_0^2 \Sigma\alpha^2 = a_1^2 - a_0a_2,\ 2a_0^3\Sigma\alpha^3 = -(a_0^2a_3 - 3a_0a_1a_2 + 2a_1^3),$$

$$12a_0^2\Sigma\alpha^2\beta^2 = a_0a_4 - 4a_1a_3 + 3a_2^2.$$

Thus seminvariants of a quantic can be treated as non-unitary-partition symmetric functions of an associated ordinary algebraic equation. This range of invariantive forms can therefore be constructed from the results of the pure algebra of symmetric functions, the weight of the form being the same as the weight of the function. In turn, by its dependence on the non-unitary-partitions of the weights, this pure algebra invokes the theory of the partitions of numbers. MacMahon's initial discovery thus made a link between the theory of invariants and one branch of the theory of numbers. As invariants can arise through continuous variation of the magnitudes which occur, while partitions are necessarily concerned with discrete magnitudes, all the cognate work established another connexion between the calculus of continuous quantity and the calculus of discontinuous quantity.

The ensuing researches had one important, almost an immediate, result. Progress in the theory of covariantive forms had, to some extent, been barred through lack of a complete mastery over the syzygies, that is, the homogeneous relations among the seminvariants. But owing to this new correspondence between seminvariants (or perpetuants, as Sylvester styled them) and the non-unitary-partition symmetric functions of the roots of an equation, the syzygies in question were transferred to the region of the selected symmetric functions. The last can be enumerated by means of the partitions; the relations, which connect them, are established by algebra of a simple character initiated by Newton: and the calculations thus became matters of pure arithmetic and algebra.

The way thus was indicated for a new algebraical proof of one theorem of fundamental importance—the finiteness in number of the aggregate of the asyzygetic concomitants of a binary quantic; it had previously been known only by Gordan's proof, which used the methods of umbral representation. By Cayley and MacMahon, among others, especially in relation to syzygies, to generating forms, and to ground-forms, the algebraic work was developed. By MacMahon himself, utilising earlier researches of Sylvester and others on the partition of numbers (and, as a happy incident, securing for publication some forgotten unedited lectures of Sylvester on the subject) a full development of partitions in general, with many ramifications and amplifications, was revealed. The work was (what, in another connexion, Sylvester had called) a new world of analysis.

Accounts of the work, while still it was in progress, will be found (though with much self-effacement) in MacMahon's presidential address to the London Mathematical Society (*Proc. Lond. Math. Soc.*, vol. 28, pp. 5-32; 1897) in 1896, and in his address as president of Section A at the Glasgow meeting of the British Association in 1901 ("British Association Report", pp. 519-528; 1901). Memoir succeeded memoir in his consecutive development of the theory over a number of years. He dealt with matters apparently so diverse as symmetric functions; differential operations and their comparative effects: partitions of numbers, unipartite and multipartite, their separations and their compositions: permutations, in a multitude of associations: Euler's Latin squares; magic squares, ancient and modern: diophantine equations and inequalities: enumerating (or generating) functions, in their persistent emergence throughout the theory. Finally, he produced a systematic account of all this work, including cognate investigations of other writers, in the treatise "Combinatory Analysis", cited earlier. It is a fitting and an abiding monument to his genius.

Nor did MacMahon disdain the lighter issues of his work. It admitted of illustrations and applications that can appeal (though, as the world estimates pure mathematics, too seldom do appeal) to those who are unversed in mathematical phraseology and mathematical conclusions. Thus he delivered an almost untechnical lecture on "Magic Squares and other Problems on a Chess-Board" as a Friday evening discourse at the Royal Institution on Feb. 14, 1902. At the same place, on the afternoons of Jan. 30 and Feb. 7, 1907, he lectured on "Standards of Weights and Measures". His interest extended to special problems such as finding the totality of ways of seating a number of married couples at a round-table dinner-party, so that each lady sits between two gentlemen and no lady is next her husband. He found relaxation and amusement in using the ideas of combinations and permutations for the construction of ingenious (yet not useless) pastimes. Thus, to select one instance, full sets of pieces of cardboard are required: all the pieces of a single set are to be of the same shape (usually triangular, or square, or hexagonal) and of the same size: they are to be coloured, each, for example, with three out of four colours, while no two are to be coloured in exactly the same way. By the adoption of definite rules for combining the pieces of a set, a large number of different forms can be obtained, each such form being a geometrical pattern. Each pattern can be repeated so as to provide a general symmetric design. The designs can be utilised in a variety of ways: for humble wall-paper, for mosaics and woven fabrics, for the refined ornament of architecture. In a small volume entitled "New Mathematical Pastimes", published in 1921 by the Cambridge University Press, he gave an account of these recreations, at once light and serious: the contents are entirely his own creation.

MacMahon's investigations extended over nearly half a century. Many in number, diverse in range, they constitute a fine contribution to his science, and they assure him an honourable place among the prominent pure mathematicians of his generation.

A. R. F.

PROF. T. BRAILSFORD ROBERTSON.

NEWS has been received of the death on Jan. 25, from septic pneumonia, of Prof. Brailsford Robertson, of the University of Adelaide. His premature death, at the comparatively early age of forty-five years, removes one of the most active and valuable workers from biochemical research, and is a very serious loss to the recently instituted movement for the more rapid application of biological knowledge to the development of animal husbandry in Australia.

Thorburn Brailsford Robertson was educated at the University of Adelaide. In 1904, attracted by the work of the late Jacques Loeb at the University of California, he went there as a research student in biology and for several years worked in close collaboration with Loeb, and eventually succeeded him as professor of biochemistry and pharmacology at Columbia University in 1916. In 1918 he was called to the chair of biochemistry at the University of Toronto in succession to Prof. A. B. MacCallum, and in 1920 he returned to Adelaide as professor of biochemistry and general physiology in succession to his father-in-law, the late Sir Edward Charles Stirling.

From 1920 until the time of his death, Prof. Brailford Robertson occupied a prominent position in Australian biological science, both in pure research and in the application of the results of research to industrial problems. He was one of the founders of the *Australian Journal of Experimental Biology and Medical Science*. When the Commonwealth Council for Scientific and Industrial Research was instituted a few years ago, he was invited to become the chief officer in charge of investigations on the nutrition of animals. To enable him to devote the major part of his time to this work, he was relieved of teaching at the University, though he continued to be a member of the Senate, so that his experience might be available in developing the school of biochemistry and physiology at the University.

Prof. Robertson was an assiduous worker and a prolific writer. He did most important work on the physical chemistry of the proteins, and later conducted long and laborious research on problems of growth and senescence. Among the problems of general physiology to which he made valuable contributions may be mentioned allelocatalysis as a factor in the multiplication of infusoria, the permeability of cells and the underlying physico-chemical principles involved in cell division. In addition to numerous papers in scientific journals, he published "The Principles of Biochemistry" and other two works, namely, "The Physical Chemistry of the Proteins" and "The Chemical Basis of Growth and Senescence", in both of which he incorporated the results of his own original work on these subjects.

Prof. Robertson had a stimulating personality, and, as a lecturer, had the gift of imparting his new enthusiasm to his audience. His death will be deeply regretted in scientific circles, especially in North America and Australia, in both of which continents he exercised a great influence in the development of biochemistry, both as a science and in its application to practical problems. The loss of his profound scientific knowledge and great experience in organisation will be a very serious blow to the work of the Commonwealth Council for Scientific and Industrial Research. J. B. O.

MR. F. P. RAMSEY.

THE death on Jan. 19 of Frank Plumpton Ramsey at the early age of twenty-six has cut short a life which bore exceptional promise of eminence in mathematics and philosophy. The elder son of Arthur Stanley Ramsey, now President of Magdalene College, and the author of well-known treatises upon subjects in applied mathematics, Frank Ramsey was born in 1903 and passed his boyhood in Cambridge. From King's College Choir School he became first a scholar of Winchester, and then a scholar of Trinity College, Cambridge: in 1923 he graduated in the first class of the Mathematical Tripos, with distinction, and in 1924 was elected to the Allen (University) Scholarship. At the time of his death he held a University lectureship in the Faculty of Mathematics, and was a fellow and director of studies at King's College, Cambridge.

It could not be expected that Ramsey's published work would fill a large number of pages; yet there is enough to prove the distinction of his mind and powers. The London Mathematical Society has printed two weighty papers, "The Foundations of Mathematics" (1926) and "On a Problem of Formal Logic" (1929). The former, written after Ramsey had become acquainted with the work of Wittgenstein, is probably his most important original production. In it he aims at presenting the general method of Whitehead and Russell in a