The Influence of Gravitation on Electromagnetic Phenomena.[1]

By Prof. E. T. WHITTAKER, F.R.S.

IN the Bakerian Lecture of 1850, Faraday described a series of experiments which he had made in searching for a connexion between gravity and electricity. "The long and constant persuasion," he said, "that all the forces of Nature are mutually dependent, having one common origin, or rather being different manifestations of one fundamental power, has made me often think upon the possibility of establishing, by experiment, a connexion between gravity and electricity, and so introducing the former into the group, the chain of which, including also magnetism, chemical force, and heat, binds so many and such varied exhibitions of force together by common relations."

The results of Faraday's experiments were negative; but, he said, "they do not shake my strong feeling of the existence of a relation between gravity and electricity, though they give no proof that such a relation exists." The proof was, indeed, not obtained until seventy years afterwards, when the measurements of photographic plates taken at the eclipse of May 1919 showed a deflexion of the rays of light from stars when the rays pass close to the gravitating mass of the sun.

The general theory of relativity, by which this deflexion was predicted, asserts that the presence of matter or energy in any region of space affects the metric in that region; as we may say, it causes a 'curvature' or 'distortion' of space, which not only determines all the gravitational effects in the region, but also has a remarkable influence on any electromagnetic phenomena which may be taking place there. This is the true solution of Faraday's problem.

It is important to notice that gravity and electricity have been brought into connexion in essentially the same way as light and electricity were brought into connexion by the Maxwellian electromagnetic theory of light, namely, by postulating that the same 'ether' transmits both kinds of actions. It is true that we do not speak much of the ether nowadays, and certainly do not regard it as a quasi-material medium filling all space; but when we endow space itself (or, in non-statical problems, space-time) with properties such as curvature, we are making it play the part of an ether. The principle that one and the same ether ought to serve for all purposes was enunciated by Faraday himself: "It is not at all unlikely," he said, "that if there be an ether, it should have other uses than simply the conveyance of radiations."

In what follows I shall make the simplifying assumption that the gravitational field is 'statical,' that is, such as would be produced by gravitating masses which are permanently at rest relative to each other, so that the curvature of space at any point does not vary with the time. In the distorted space of this fixed gravitational field, I suppose an electromagnetic field (either statical, or varying with the time) to exist; strictly speaking, the electromagnetic field has itself a gravitational effect, that is, it changes the metric everywhere; but this effect is, in general, small, and we shall treat the ideal case in which it is ignored, so we shall suppose the metric to be simply that of the gravitational field originally postulated.

The problem before us, therefore, is to study the existence and propagation of electromagnetic fields in a medium whose properties (that is, the distortions of space) vary from point to point, and this naturally suggests a comparison with the Maxwellian theory of electromagnetic fields in a medium the specific inductive capacity and magnetic permeability of which vary from point to point. Do the effects of the distortion of space resemble in any way the effects of a variable dielectric constant and permeability?

The answer is in the affirmative, though the resemblance is not quite perfect. In a gravitational field there are eight partial differential equations which have exactly the same form as the usual Maxwell's equations, but in place of the three simple linear equations which connect the components of the electric displacement of the Maxwellian theory with the components of the electric force, and the three equations which connect the components of the magnetic induction with the components of the magnetic force, we now have six linear equations which express six of the twelve components in terms of all the six others.

Thus, from the mathematical point of view, the problem is similar to that of the Maxwellian electromagnetic field in a medium the dielectric constant and magnetic permeability of which have a kind of six-fold æolotropy. A prophetic adumbration of all this is to be found in a remarkable sentence written by FitzGerald so long ago as 1894: "Gravity is probably due to a change of structure of the ether, produced by the presence of matter."

To learn what happens we must solve these equations, and, as a first and simplest case, let us suppose that the electromagnetic disturbance is a ray of light. In this case it is not necessary to obtain the complete solution of the partial differential equations, since we can make use of the theorem that "a ray of light is a null geodesic of space-time." Let us, then, suppose that the gravitational field is due to a single gravitating mass, which we may call the 'sun,' and let us find the null geodesics of this field, which will be the paths of rays of light in the field of a single gravitating centre. We find that a ray of light which comes from infinity and does not pass too near the 'sun' is simply deflected through a small angle, in the same way as the light from stars was actually found to be deflected in the eclipse photographs. But if it is aimed almost directly at the mass (which, it must be remembered, we are supposing to be collected in a point-centre), much more interesting things may happen. Thus if a certain constant

[1] From a lecture delivered to the London Mathematical Society.

depending on the initial conditions has a particular value, we obtain light-rays which are spirally asymptotic to a certain circle surrounding the 'sun'; one type of ray represents light which, coming from infinity towards the mass, is 'captured' by it, and never gets away again, but circles round it for ever; another type of ray, on the other hand, represents luminous energy which is, and always has been, imprisoned in the immediate neighbourhood of the mass.

These phenomena cannot be observed in the case of our actual sun, because its mass is not sufficiently concentrated: the sun's bulk prevents the light rays from getting close enough to its centre; but it seems conceivable that, at the nucleus of an atom, we may have a concentration of mass into a space so small that the capture of light by an intense gravitational field may be realised.

To pursue this matter somewhat further, let us consider the field round a point-mass. If we draw round this mass a circle, the length of the perimeter of the circle may be denoted by $2\pi R$, a definition which determines the physical meaning of the quantity R. But the normal distance between two adjacent circles, of perimeters $2\pi R$ and $2\pi(R-\delta R)$, is *not* δR, but δR multiplied by a multiplier which increases indefinitely as R approaches a certain value a which depends on the sun's mass: that is to say, as we approach the circle of perimeter $2\pi a$ from outside, we find greater and greater difficulty in making any headway; we have to travel a very great distance in order to pass from one of these circles to another just inside it, the perimeter of which differs from it only very slightly, and we can never actually attain to the circle with perimeter $2\pi a$.

If, then, we consider a ray of light coming from infinity and travelling directly towards the point-mass, the velocity of the light will always be c; but when it begins to approach the circle of perimeter $2\pi a$, this velocity will only be sufficient to carry it onwards very slowly, if we measure its progress by the rate of diminution of perimeter of the circles it cuts through, and it can never, in any time however great, get nearer to the mass than the circle of perimeter $2\pi a$.

Thus, although the light is actually travelling for ever with its usual velocity, it remains permanently in the neighbourhood of the point-mass. The capture and imprisonment of radiation by the intense gravitational field surrounding a point-mass is a remarkable theoretical possibility, markedly different from anything in pre-relativity physics.

Let us now leave the consideration of light rays and pass on to other kinds of electromagnetic phenomena. The mathematical difficulties here are greater, since we now cannot avoid the partial differential equations; but they can be solved in many cases, provided that we take the simplest possible type of gravitational field, which may be arrived at in the following way. Consider the field due to a single gravitating centre, and, fixing our attention on the neighbourhood of a point O, suppose the gravitating centre to be removed to a very great distance from O, while its mass is increased, so that the attractive force at O (to use the language of the older physics) remains finite and equal to g; then we obtain what we may call a *quasi-uniform* gravitational field. In the neighbourhood of O it is essentially the 'uniform gravitational field' of the old physics.

Let us now consider the shape of the equipotential surfaces in a quasi-uniform gravitational field, due to a single electric charge at (say) the origin. We find that these equipotential surfaces are a family of coaxal spheres, having one limiting point at the origin. Thus the difference which a quasi-uniform gravitational field makes to the equipotential surfaces of a single electric charge is that, instead of being concentric spheres, they become coaxal spheres—they become more crowded together on one side of the charge, and less crowded on the other. The effect is exactly the same as if we supposed that, instead of having a gravitational distortion of space, we had the specific inductive capacity and magnetic permeability of the medium each varying as we move in the gravitational field, so that the medium is stratified at right angles to the direction of gravitation.

From these calculations we can deduce a physical result of some interest. Suppose we have an electric charge at rest at the origin, and suppose we have initially a quasi-uniform gravitational field in some particular direction, and that we then reverse this so as to have a quasi-uniform gravitational field in the opposite direction, and then reverse back to the original state of things, and so on. At each reversal the electric equipotential surfaces will change from being a family of coaxal spheres with their second limiting point in one direction, to a family of coaxal spheres having their second limiting point in the opposite direction. But this regular alternation of the electric field must set up radiation, just as the alternation of the electric field in a Hertzian oscillator does: and therefore an electron at rest in a varying gravitational field will, in general, emit radiation.

The knowledge that a motionless electron may radiate, while (as a natural consequence) an accelerated electron does not necessarily radiate, in a gravitational field, may prove useful in accounting for the behaviour of electrons in atoms.

Let us now pass on to the case when the gravitational field is that due to a single gravitating mass at a point, so that we have Schwarzschild's metric. The solution which represents the potential of a single electric charge, and therefore corresponds to the $1/R$ of the ordinary theory, has recently been discovered by Mr. Copson.[2]

The form of the equipotential surfaces is remarkable. Very near the electric charge they are, of course, practically spheres the centre of which is at the charge. But as we get farther from the charge and nearer to the gravitating mass, the equipotentials behave as if they were repelled by the mass, so that eventually they become concave to it (and therefore convex towards the charge!), the mass being in a cup-shaped depression in the

[2] *Proc. Roy. Soc.*, A, **118**, 184; 1928.

equipotential surface. Eventually we reach an equipotential which consists of two closed surfaces touching each other, one (the smaller one) enclosing the gravitating mass but not enclosing the charge, while the other (the larger one) encloses this smaller surface and also encloses the charge. Beyond this, again, we have a series of simple closed surfaces, each of which encloses all the earlier members of the family.[3]

The electric field is precisely the same as would be obtained, in the ordinary electrostatics, by supposing that the specific inductive capacity and magnetic permeability of the medium vary in a certain way with the distance from the gravitating mass.

Solutions of the fundamental equations have been found which represent more complex fields than those I have described: but they are perhaps not well suited for description in a lecture,[4] and I will therefore conclude with a remark about energy. In a *statical* gravitational field, electromagnetic energy is a scalar quantity, so we can calculate the 'total electromagnetic energy' contained in a specified region of three-dimensional space by integrating the amounts of energy contained in the sub-regions into which the region may be divided; and the conservation of electromagnetic energy holds. In *non-statical* gravitational fields these theorems are no longer true, since energy is not then a scalar quantity; the energy in one region is different in kind from the energy in another region, just as momenta in different directions are different in kind from each other.

[3] A figure is given in Mr. Copson's paper, loc. cit.

[4] Reference may be made to *Proc. Roy. Soc.*, A, **116**, 720; 1927.

Heirlooms of Industry in the Science Museum.[1]

By Col. Sir HENRY G. LYONS, F.R.S.

THE idea of a general museum of science is only seventy-five years old, and was due to the Prince Consort, who, after the Great Exhibition of 1851, urged the formation of an institution which would extend the influence of science and art on productive industry. From this proposal arose the Science and Art Department with, as an essential part of it, the South Kensington Museum, the dual activities of which are now represented by the Science Museum and by the Victoria and Albert Museum respectively.

For a long time previous to this, scientific instruments, pieces of apparatus, mechanical devices, and such like had been preserved in many places, but a museum designed to illustrate the influence of science on technical development did not exist; even the Museum of the Conservatoire national des Arts et Métiers in Paris, which dates from the end of the eighteenth century, was, and still is, primarily the teaching collection of the Conservatoire. Recently technical museums having similar aims have been established at Munich in 1903, and at Vienna in 1919; others illustrating special industries are to be found in many cities.

While no one will dispute the utility of technical museums, it must be admitted that they fall far short of art museums in the attractiveness of the objects which they contain, and this fundamental difference affects every stage of museum arrangement. For the objects in a technical museum to be interesting, something of their history and their purpose, the part which they have played in the age-long development of the branch of industry to which they belong, must be known to the visitor, and to this end carefully edited labels are essential; important objects may be so displayed that the internal working parts can be seen, and their purpose understood; coloured diagrams and specially arranged illumination may be employed; models, etc., may be shown in motion; others may be so arranged that they can be set in motion by the visitor; all this being done with the object of supplying essential information in a form intelligible to the general visitor, with additional technical description for the specialist.

The dominating principle in such a museum must be to illustrate development. Everything, whether it be an industry, or a group of related objects, or a type of tool, is shown so as to emphasise the successive stages of development which have been traversed from early crude forms which sufficed in the days of hand labour through various grades of slow improvement to the rapid advances of modern times so effectively aided by steam and electricity. In some branches of human activity it is instructive to show a few examples from the times of the earliest civilisation, and from the handiwork of primitive races who exist to-day. In this way the story of time measurement, of various hand tools, and of land and water transport for example, can be illustrated far more effectively and attractively than if the exhibits were restricted to those dating from the last few centuries.

To take the case of an industry, aeronautics furnishes a convenient example of rapid development, for air transport has grown from its first experimental stages to an important branch of world communication within the lifetime of many of us. The series exhibited in the Science Museum begins with the first power-driven model aeroplane which John Stringfellow constructed in 1848, and which achieved a free flight of forty yards. Later on experiments in 'gliding flight' were of great importance in providing information on many points in aerodynamics, and Otto Lilienthal was its greatest exponent. One of his gliding machines of 1896 is exhibited. During the next seven years Prof. Samuel P. Langley, of the Smithsonian Institution at Washington, designed and constructed a man-carrying tandem monoplane, of which a model is on exhibition, but it failed to make a successful flight when tried in 1903.

Inspired by the experiments of Lilienthal in

[1] Synopsis of a discourse delivered at the Royal Institution on Friday, April 20.