Philosophical Foundations of Quantum Theory.[1]

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THE development of physics in the last decades has repeatedly raised epistemological questions of fundamental importance. Thus in the theory of relativity the problem of space and time received at least a temporary clarification. New questions have arisen in connexion with the quantum theory, and particularly the question of the existence of causal laws in elementary physical processes. Is the condition of an atomic system completely determined, or are there gaps in its determination?

Physicists no longer doubt that this question of the existence of a complete causal determination can only be settled by experience, and that causality is not an *a priori* necessity of thought. Certainly some degree of determinism is an essential condition for the possibility of physical science, as it is for any ordered and intelligible existence; and fortunately we have, if we confine ourselves to macroscopic phenomena, an apparently universal and trustworthy determinism. But for atomic phenomena this implies only a statistical determinism. The question of the causal necessity of the laws of an individual atom thus remains.

Before we attack this point it will be useful to give the notion of determinism a more precise consideration. The physicist cannot be satisfied with the approximate idea that we have of the meaning of this word. Nor is he interested in the metaphysical significance that some philosophers give it. For the physicist the definition of causality or determinism means the specification of conditions by which its existence may be experimentally established. This shows that the physical definition must continually change in accordance with the basis of our theories and facts and experimental methods. Let us consider first the rôle of causality in the classical physics of the field.

The contention of this classical physics is that the physical world may be described—and we use the word describe in a more or less purely geographical sense—by the specification of certain measurable quantities—fields, potentials, etc.—for every point of a four-dimensional region of space-time; and then the causal determinism consists in this. Let us consider a bounded volume of the space—say a parallelepiped. We shall not consider what modifications would be introduced into these considerations by taking complete account of the relativistic relations of space and time; but this would, of course, raise no difficulty. At a certain time—for example, at eleven o'clock—let the physical conditions inside the box be completely known and measured. Further, let the physical conditions of the entire surface of the box be specified from eleven until twelve o'clock. The physical phenomena in the whole box will then be uniquely determined from eleven to twelve o'clock. Thus, if we at any other time and place repeated the experiment, with the same initial conditions and the same course for the surface conditions, all of the phenomena inside the box would automatically be reproduced. Within a certain period of time—of the order of magnitude of the dimensions of the box divided by the velocity of light—the phenomena inside the box are independent of those on the surface.

These assertions are susceptible of experimental proof. Of course, we have to suppose at the beginning that the initial state within the box is not so complicated that its complete physical investigation would be entirely impossible. Thus we should have to exclude the case that there is a living organism in the box; for the notion that one could measure exactly the physical conditions is in this case not in accord with the experimental practicability. Thus the notion of determinism must be formulated differently for biology and for physics.

Let us therefore confine ourselves to a consideration of physical determinism. We may remark that this determinism is of an extraordinary kind; it is not at all equivalent to the mere existence of physical laws—the existence of mathematical relations between physical quantities. Moreover, we have here a peculiar asymmetry between the special and temporal co-ordinates; by this principle of determinism two temporally separate regions may influence each other physically; two spatially separated regions cannot.

The theoretical justification of this determinism arises from two circumstances. We shall mention these here without entering upon a mathematical proof that they really furnish this justification. In the first place, the physical laws—that is, the mathematical relations between the components of the field—are differential equations, and, in fact, to a first approximation, principally linear partial differential equations of the second order. In the second place, for the simplest four-dimensional geometry, in which the Pythagorean theorem remains valid, one must use, not the time itself, but the imaginary time, as fourth co-ordinate. If, instead of this, the four-dimensional manifold had four real dimensions, and the differential equation of physics remained unchanged, then we should have a much more complete determinism: one would then be able to deduce the state of any region of space-time from an accurate knowledge of any other specific region. If, on the other hand, the world had two real and two imaginary dimensions, there would be no determinism left. For in this case it would be possible for new motions suddenly to arise inside a box, even though there were no cause for them either within the box or at the boundary. This, then, is the significance of determinism for the physics of the field. It is not itself

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a natural law ; the natural laws are the differential equations that lie at the basis of the physical field. It is a mathematical conclusion from these natural laws, a theorem from the mathematical theory of hyperbolic differential equations which has been applied to the laws of physics.

It is on this account that one must be prepared to lose this determinism in the transition from classical physics to the quantum theory. For just those physical assumptions are here discarded which we have noted as theoretical support for the existence of determinism. Even the *description* of 'physical reality' cannot, as we now know, be given in terms taken from classical physics. Physical quantities are *not* continuously propagated through space ; physical motions are not invariably continuous ; there are elementary discontinuities, there are *quantum jumps*. What remains of determinism is not necessarily more than statistical. If we work with a great many similar atoms, or repeat very often experiments with a few, then we always get a result in agreement with the principle of determinism. We have seen before that physical determinism and physical laws were not co-extensive. We must, therefore, remark that in this case what we have said for determinism holds also for all physical laws. All we know at present are laws that are essentially statistical.

In recent times important advances have been made in the discovery of these laws. One can now, for example, compute (in principle) the spectrum connected with the motion of electrons in an atom with the same assurance as, on classical dynamics, one could calculate the motions of the planets. But in spite of the analogy between the calculations, there is an important difference in the interpretation of their results. The classical calculation gives us information about our specific system of planets. The quantum theoretical calculation does not, in general, tell us anything about a single atom, but only about the mean properties of an assembly of similar atoms. One can, it is true, calculate the energy of a single atom in a certain state on the quantum theory. But that is only because the energy is the same for all atoms in this state, and the individual energy coincides with the mean. But if we consider the behaviour of the atom under external influences—*e.g.* incident light or electronic collision—we get a result that cannot be interpreted as showing that, for specific values of the phases, specific phenomena occur. We can only interpret the results of the calculation as follows : there is an assignable probability that the atom will do one thing, and an assignable probability that it will do any other.

We have a similar situation in optics. Classical optical theory yields, it is true, all interference phenomena in perfect accord with experiment. But the calculated intensity of light at a given point does not represent the actual intensity. The classical wave-field clearly only gives the probability that a quantum will reach the point. Moreover, one can find waves accompanying a material corpuscular beam which in some respects bear the same relation to the corpuscles as the light waves to the quanta. Here again we see the purely statistical nature of the present quantum theoretical laws.

We shall therefore direct our attention, not to the discrete, discontinuous details, but to the corresponding probabilities. The introduction of these probabilities brings us formally back to continuous variables, and thus, in a very important respect, to classical terms. We are therefore led to suppose that there is a principle of determinism for these continuous probabilities which is not very different from the classical principle. This is, in fact, the case, but in a somewhat more abstract way than in the classical theory.

It is known that Schrödinger, independently and by methods of his own, was able to give a formulation of the quantum theory which turned out to be mathematically equivalent to the matrix theory based on Heisenberg's ideas. He thus discovered mathematical relationships in the quantum theory, which had not before been explicitly developed. These were, it is true, implicitly involved in the matrix theory ; but their formulation represents an important addition to quantum mechanics.

In connexion with these formulæ, Schrödinger also tried to develop a new physical basis for the quantum theory. His interpretation differs fundamentally from that of Planck, Bohr, and Einstein, from the classical quantum theory based on stationary states and quantum jumps. In it he tried to return to quasi-classical conceptions, in which there were no longer any discontinuities, and in which, therefore, the classical principle of determinism was still valid. All other scientific workers, however, who had taken part in the development of quantum mechanics, were unable to accept these speculations of Schrödinger. They were sure that the new conceptions would have to be interpreted physically in close analogy with the older notions of stationary states and quantum jumps, and with Heisenberg's theory ; and that Schrödinger's relations would, like those of the matrix theory, have to be interpreted statistically. A particularly clear and satisfactory formulation of the statistical interpretation of Schrödinger's theory has been given by Born, and in what follows I shall base my argument on this.

The essential purport of Schrödinger's theory is this : that quantum mechanical laws, which were given in the matrix theory as a system of infinitely many equations with infinitely many unknowns, can instead be expressed by quite ordinary differential equations. Formally, this takes us back very close to the classical theory. Born's answer to the question as to how it is possible to represent anything in the discontinuous confusion of quantised atomic processes by differential equations, is that the function which is to satisfy the differential equation is a probability.

We shall now consider more closely this probability function, and try to make clear the analogy with the classical situation. Consider a mechanical system of two particles and six degrees of freedom ; let the continuous co-ordinates of the particles be x_1, x_2 to z_1, z_2. We now construct a space more or less like the classical phase space, but with only half as many dimensions: a configuration space. In our example it is a six-dimensional space with the co-ordinates x_1 to z_2. We can represent the state of the system when it has specified co-ordinates and arbitrary momenta by a point in this space, which we shall call the system-point. Classically the system-point would describe a certain trajectory. But we cannot know beforehand, if we observe it at a given time at a given place in the space, how it will move, for the system-point only tells us the co-ordinates, not the momenta, of the two particles. All one can determine is the probability that the point will move in a given direction.

In the classical theory we can translate this statistical prediction into an exact one, since we can observe not merely the position but also the velocity of the system-point. But, according to Pauli, this is just the point where quantum and classical theory differ. If we can observe the co-ordinates of a quantum mechanical system—and here we use *co-ordinate* in a generalised sense, so as to include, say, the energy or the quantum numbers—then the momenta conjugate to these co-ordinates are intrinsically not observable. All we can do, therefore, is to take over the statistical problem from classical mechanics, and probably derive the solution from Schrödinger's differential equation. I say *probably*, because considerations which follow out these speculations are not yet completed.

The following question, however, which is closely connected with the one we have been considering, *can* be regarded as solved by the work of Born and Pauli. Suppose we know the energy and quantum numbers of our system—or, more generally, suppose we know the probability that the system is in any of the stationary states; what is the probability that the system-point is at a given place in the configuration space? We can answer this question at once if we know the Schrödinger wave function.

This Schrödinger function—a function of the six variables x_1 to z_2 and the time—satisfies Schrödinger's fundamental differential equation. For this probability function we can again formulate a principle of determinism. For this, of course, we have to take a box, not in ordinary space, but in the six-dimensional configuration space. Then the principle is precisely the same as in the classical theory, except that in place of the measurement of the field inside and on the boundary of the box we must now write the measurement of the Schrödinger probability function.

To recapitulate: classical physics described the world in terms of quantities continuously propagated in space and time. The quantum theory describes the world in terms of an abstract, many-dimensional configuration space, and the number of dimensions is proportional to the total number of particles in the world. In this abstract space we have again the propagation of continuous quantities; but these no longer tell us directly about the single atomic phenomenon, but rather about the probabilities of quantum processes. Determinism—not as a metaphysical distinction from chance, but in the physical sense explained above—has the same formal validity in both theories.

Of course, one can transform the quantum theoretical laws back to ordinary space, but their form is then very complicated, for the abstract space leads to the most suitable formulation of the problem. But one can still say that the complicated 3-dimensional predictions justify, roughly, what I said before—that in the mean the old 3-dimensional determinism still holds.

We have seen how it is possible, by the use of averages and probabilities, to eliminate the elementary discontinuities in physical processes, and to find relations which can be formulated mathematically by the customary methods of classical physics, methods adapted to the study of intrinsically continuous quantities. In this respect quantum mechanics constitutes a more precise version of Bohr's correspondence principle. Bohr always (it will be remembered), even in the zenith of our belief in integers, insisted that we try to establish a formal analogy with classical theory by a consideration of mean values.

Now, however, we shall return to the problem of the discontinuous elementary phenomena; we shall consider the question of how much we can say about these phenomena, granted that we can find the solution of any problem in averages, whatever their formulation. The answer to this question is not nearly so simple as one might expect; and I should be guilty of a very superficial treatment of my subject if I were not at least to point out some of the difficulties that occur in connexion with it.

Let us first of all examine the matter from the empirical point of view. One might suppose that experiments would in no case give us anything but average values. An interesting lecture given by Prof. Zernicke last summer on the Brownian movement, and in particular on the researches of Ising in Sweden, showed clearly the impassable limits to an improvement of the technique of physical measurement. It is impossible to increase the accuracy of a galvanometer, for example, beyond a certain assignable limit; it is impossible because of the Brownian movement in all parts of the apparatus. The needle, the fibres, the housing, the surrounding air, all consist of atoms in irregular thermal agitation; and the current that passes through the galvanometer consists of electrons, and therefore shows irregular variations of intensities, which can only be statistically computed and limit the efficiency of the instrument in an analogous way. When we remember that this is the case with all our apparatus, and that it all 'rattles about' in this way, we may be tempted to think that the experimentalist is quite as incapable of observing elementary processes as the quantum theorist is of predicting them. But there is a drastic method of avoiding Brownian movement. The theorist gives the simple recipe: make the experiments at the absolute zero. Luckily, experimentalists have discovered an equivalent but less uncomfortable way. By working with particles which have a vast store of energy, *e.g.* a fast α-particle, they make the thermal agitation of the atoms negligible. And we can, in fact, largely because of the work of C. T. R. Wilson, actually observe the fate of a single α-particle, follow its trajectory, and determine the moment when the trajectory ends in a quantum jump.

The time of a single quantum jump is thus under certain conditions a measurable quantity. What predictions can our theory make on this point? The most obvious answer is that the theory only gives averages, and can tell us, on the average, how many quantum jumps will occur in any interval of time. Thus, we must conclude, the theory gives the probability that a jump will occur at a given moment; and thus, so we might be led to conclude, the exact moment is indeterminate, and all we have is a probability for the jump. But this last conclusion does not necessarily follow from the preceding one; it is an additional hypothesis. It is this hypothesis which Bohr, Kramers, and Slater tried to carry out in their theory of radiation. They realised quite clearly that this hypothesis must leave the conservation of energy as only a statistical theorem. This conclusion, of course, was disproved by the beautiful experiments of Geiger and Bothe and of Compton. We can now assert that if an atom emits light, and that if this light is propagated, unhindered by interference, to another atom, where it is absorbed, then the quantum jump of the absorbing atom occurs after a time which corresponds exactly to the distance between the atoms. Thus we see that, in some cases at least, the time of a quantum jump is determined.

One might be tempted to say: the time is determined in so far as its determination is required for conservation of energy—and no further. But this two-sided explanation is too indefinite to be of any use in complicated cases, *e.g.* where interference occurs. Another method of overcoming this difficulty was tried some time ago by Wentzel: since in our example the absorption is fully determined by the preceding emission, we could regard the two processes together as a single quantum process, and then hope that such processes would be statistically independent of each other. But this way, too, does not seem to lead to any simple formulation.

It is thus very significant that in Pauli's abovementioned formulation nothing is said about the probability of a transition—for we saw that this could not lead to independent probabilities. What the theory does specify is the probability that the system-point be at a given place in the configuration space. One might therefore hope that these considerations would lead us to independent elementary physical probabilities.

Although we can in principle compute all probabilities on the quantum theory, a very serious problem still remains unsolved. For definiteness let us take a simple example. Let us throw two dice; and let us observe empirically that a 1 and a 3 occur together just as often as a 4 and a 5, and twice as often as two 2's, and so on. Now if we had a theory which made it possible to compute these probabilities in some very complicated and abstract way, we might be satisfied. But we are really only satisfied when we can reduce the theory to this form; for each die each of the six faces is equally probable; and the dice are statistically independent. Only when we see this do we feel that we really understand the matter.

Now, for the dice we clearly should never think of using any other theory than the one just given. But in the quantum theory the matter is different: we can at present compute all probabilities; but we cannot understand any of them. We could only say that we understood them if we had translated the calculations in configuration space into the following terms: In some cases there is no condition on what happens; either this or that can happen; they are equally likely, and what happens in one case has nothing to do with what happens in others.

In other words, we must reduce the quantum theoretical probabilities to independent elementary probabilities. Only then can we say that we really understand the laws; and only then can we tell under what conditions the time of a transition is determined. Only then can we know exactly what is causally determined, and what is left to chance.

In conclusion, let me try to bring out one more point. We have just been taking for granted that the future analysis of quantum theoretical probabilities would lead to the result that certain elementary processes were not determinate, and could happen equally probably in a variety of ways. But in fact that is not at all self-evident. The circumstance that quantum laws are laws of averages, and can only be applied statistically to specific elementary processes, is not a conclusive proof that the elementary laws themselves can only be put in terms of probability.

We can thus put in this final form our question, "Does modern physics recognise any complete determinism?"—a question which we have seen to split up into several distinct ones. Will the elementary laws for which we are looking be laws of determinism or of probability? Will it ever happen that the time of a quantum jump is undetermined?

Probably we shall find that an incomplete determinism, a certain element of pure chance, is intrinsic in these elementary physical laws. But, as I have said, a trustworthy decision will only be possible after a further analysis of quantum mechanics on the lines laid down by Born and Pauli. Perhaps I might add that pertinent considerations have been recently carried through in Copenhagen, and here in Göttingen, in what, I think, is a very promising way.