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The Prediction of Eclipses.

By Dr. L. J. Comrie.

THE fact that astronomers can predict, many years in advance, the time of a solar eclipse to within a few seconds, and the region on the earth's surface in which it will be visible to within a mile or so, will perhaps surprise the layman. Yet attention was directed to the forthcoming eclipse of June 29 more than half a century ago by the Rev. S. J. Johnson in his " Eclipses Past and Future."

These advance predictions are possible because eclipses pass through a definite cycle, or repeat themselves after an interval of 18 years 11⅓ days. To explain the reason for this, consider the conditions that lead to an eclipse. It is well known that the earth moves round the sun in a nearly fixed plane called the ecliptic. As seen from the earth the apparent motion of the sun in the heavens is along the great circle which represents the intersection of the ecliptic plane and the celestial sphere, or the ecliptic circle. If the motion of the moon round the earth were in the same plane, then once in every lunar month of 29½ days the moon would be in line with the sun, and an eclipse would result. Actually the plane of the moon's orbit is inclined to the plane of the ecliptic at an angle of 5°. This means that the great circle representing the moon's orbit crosses the ecliptic at two opposite points or nodes, while half-way between the nodes the nearest possible approach of the two bodies is 5°.

Should the sun happen to be within a certain limiting distance from either of these nodes at the time the moon crosses the ecliptic, there will be an eclipse, either partial or total. If the nodes remained fixed on the ecliptic, eclipses would take place at the same two seasons each year. Actually the nodes make a complete circuit of the ecliptic in a backward or retrograde direction in a little less than nineteen years, so that the sun, in its annual course through the ecliptic, returns to the same node, not in 365·24 days, but in 346·62 days, a period called the eclipse year. Nineteen of these periods equal 6585·78 days. The interval between successive conjunctions of the sun and moon, or, in other words, the interval between successive new moons, is called the synodical month, and is 29·53 days. 223 of these lunations contain 6585·32 days. That is to say, if the sun and moon at any given moment are in conjunction at or near a node, so that an eclipse is in progress, then after 6585·32 days they will again be in conjunction and in the immediate neighbourhood of a node, so that another eclipse will occur.

The circumstances of an eclipse vary considerably with the distance of the moon from the earth—from an annular eclipse when the moon is at its maximum distance of about 257,000 miles to a long total eclipse of perhaps six or seven minutes' duration when the distance is as small as 223,000 miles. The interval between successive returns to the same distance from the earth, or the anomalistic month, is 27·55 days, and 239 of these intervals are 6585·54 days. Thus not only is there a repetition of the eclipse after 6585 days, but also it will occur under practically the same conditions.

This most useful period of recurrence was known to the Chaldeans as the Saros, and formed the basis of their very successful eclipse predictions. It is still used for the purpose of fixing the dates on which eclipses will occur, although the details of the eclipses are obtained by more refined methods.

One question must be answered. Why was not a total eclipse visible in England eighteen years ago, in 1909 ? The reason is that 223 lunations exceed 6585 days by about 8 hours, and in that time the earth has rotated, so that successive corresponding eclipses occur, on the average, 120° of longitude farther to the west.

Mention should be made here of Oppolzer's celebrated " Canon der Finsternisse " (Vienna, 1887), which gives particulars of all eclipses between 1207 B.C. and A.D. 2161, together with maps showing the central lines of total eclipses. An inspection of these maps indicates that the eclipse of 1999 will be total in Cornwall only, and that the next total eclipse visible in England in 2135 is the first of a group of four that will be seen in the course of 25 years, namely, in 2135, 2142, 2151, and 2160.

The accurate prediction of a solar eclipse is dependent ultimately on accurate predictions of the positions of the sun and moon. These bodies have been carefully observed with meridian circles for more than two centuries. The principal object in founding the Royal Observatory at Greenwich was the making of these observations, and to this day unrelaxing efforts are made to observe every meridian passage. In the hands of those masters of celestial mechanics, Delaunay, Hansen, Leverrier, Newcomb, Hill, and Brown, these observations and Newton's gravitational theory have led to tables from which the positions of the sun and moon for any date in historical times, or for centuries to come, may be found. Even the eclipses recorded by the ancients have contributed to these tables, for it is evident that the tables should reproduce the eclipses as observed.

The tables used by the "Nautical Almanac" are Newcomb's "Tables of the Sun" and Brown's "Tables of the Moon." The latter is a ponderous tome, too heavy, when bound, to be accepted by the British Post Office! Its two million figures, printed at Cambridge for the Oxford University Press, are the life-work of an English-born professor living in the United States. It requires the continuous efforts of two highly skilled computers to produce from these tables the hourly ephemeris of the moon given annually in the "Nautical Almanac."

The problem of predicting the circumstances of an eclipse for a given point on a non-spherical rotating earth would at first sight seem hopelessly difficult. But the classical solution offered a century ago by Bessel, and later most ably expounded by Chauvenet, has, by the simplicity, beauty, and rigour of its conceptions, lived to this day.

The movements of the sun and moon are expressed by means of suitable rectangular co-ordinates on a fundamental plane through the centre of the earth, and at right angles to the line joining their centres. The shadow of the moon is a cone, the intersection of which with the fundamental plane is a circle. Upon this same plane the position of the observer is projected orthographically, and the projected distance from the origin, which is the centre of the earth, is resolved into components parallel to the previously chosen axes.

The co-ordinates x and y of the moon's centre, which are also the co-ordinates of the centre of the shadow, and those of the observer ξ and η, will be the same on a plane through the observer and parallel to the fundamental plane, but the radius of the shadow-cone, L, which is readily determined, will be different. The fundamental equation of eclipse prediction simply expresses the condition that when an eclipse is beginning or ending the observer is on the edge of the shadow cone, or his distance from the centre of the shadow cone is equal to its radius. Symbolically,

$$(x-\xi)^2+(y-\eta)^2=L^2.$$

The two times when this quadratic equation is satisfied represent the beginning and ending of an eclipse.

The quantities x and y, the dimensions of the shadow cone, and other functions which are independent of the position of the observer, are tabulated in the "Nautical Almanac" for each eclipse as Besselian elements. With the aid of these elements, complete predictions for any given place can be made in a few hours.

The difficulties which prevent the making of perfect predictions must now be reviewed. First, the diameters of the sun and moon. When a bright body is projected on a dark background it appears to be larger than its true size—a phenomenon known as irradiation. Hence the diameter of the moon as usually measured has to be considerably reduced for eclipse purposes; in fact, the so-called eclipse diameter, which is used in predictions, has been determined from eclipse observations alone. Further, the limb of the moon is irregular, owing to the presence of lunar mountains; on this account alone an exact prediction cannot be made, for a valley 1000 feet deep could affect the time of eclipse by a second or more, especially if the observer were near the northern or southern limit of totality.

Another difficulty lies in the unavoidable errors of the solar and lunar tables. These arise partly from the fact that some of the quantities required in their construction, such as, for example, the masses of perturbing planets, are exceedingly difficult to determine, even from a prolonged series of observations. Another contributory cause is the fact that there appear to be some unknown influences at work. Prof. E. W. Brown, formerly a pupil of the illustrious George Darwin, in the preface to his "Tables of the Moon," says: "While many efforts have been made in the past to represent the motion of the moon by gravitational theory alone, it is now admitted that this cannot be done completely. . . . There are oscillating differences which do not correspond to any theoretical gravitational terms. . . . The causes of these differences . . . are matters of conjecture.[1] . . . Still more puzzling are certain oscillations with smaller amplitudes and shorter periods. . . . All that can be done is to make an estimate . . . from the observations of the past few years whenever it is desirable to predict the position of the moon with high accuracy, as in the case of an eclipse of the sun, and alter the values obtained from the Tables accordingly."

When the coming eclipse was first accurately predicted three years ago, a correction of +7″·0 was applied to the mean longitude of the moon as derived from Brown's "Tables," but, strangely, no correction was applied to the position of the sun. The Astronomer Royal, Sir Frank Dyson, has quoted the corrections to the longitude of the sun and moon derived from recent Greenwich observations as +1″·5 and +6″·5 respectively. When these corrections replace those formerly used, the effect is very slight; it amounts to a displacement of the central line and the zone of totality as shown on the Ordnance Survey Eclipse Map, the data for which were computed from the original elements, by just one mile in a north-westerly direction.

[1] Since the above was written, in 1918, several astronomers have suggested that the earth's period of revolution is variable, and have adduced evidence of a correlation between the anomalies in the motions of the Sun, Moon, Mercury, Venus, and Mars.

The residual uncertainty, after the application of these corrections, should be less than half a mile in the case of the central line, and not more than a mile in the case of the northern and southern limits of totality.

The co-ordinates of the central line, and the circumstances of the eclipse along this line, are given in the table below:

G.M.T.	Longitude.	Latitude.	Sun's Altitude.	Sun's Azimuth.	Duration.
	° ′	° ′	°	°	s
5h 23m 0s	+4 55·0	52 32·1	9·8	64·2	20·9
10	4 35·6	52 43·3	10·1	64·5	21·2
20	4 16·7	52 54·3	10·3	64·8	21·5
30	3 58·3	53 5·0	10·6	65·1	21·7
40	3 40·3	53 15·5	10·9	65·4	22·0
50	3 22·8	53 25·8	11·1	65·7	22·3
5 24 0	+3 5·7	53 35·8	11·4	66·0	22·6
10	2 48·9	53 45·7	11·6	66·3	22·9
20	2 32·5	53 55·4	11·8	66·5	23·2
30	2 16·4	54 4·9	12·1	66·8	23·5
40	2 0·6	54 14·2	12·3	67·1	23·7
50	1 45·1	54 23·4	12·5	67·3	23·9
5 25 0	+1 29·8	54 32·5	12·7	67·6	24·2
10	1 14·8	54 41·5	12·9	67·8	24·4
20	1 0·0	54 50·3	13·2	68·1	24·6
5 25 30	+0 45·5	54 59·0	13·4	68·3	24·9

The Recurrence of Solar Eclipses.

By Dr. J. Jackson.

AMONGST the most remarkable of discoveries made by ancient astronomers was that of the recurrence of eclipses at intervals of 18 years and 10 or 11 days. We have no knowledge of the discoverer of this period, known as the Saros, but it was certainly known to the Chaldeans. In view of the irregularities of the early calendar, such a discovery must have presented great difficulties. The fact that the interval has an odd third of a day, so that the region of visibility of an eclipse is shifted about 120° in longitude at each return, greatly increases the difficulties of discovery, and it is possible that a period three times as long as the Saros was first discovered. As the area of the earth from which an eclipse can be seen extends over a large arc in longitude, it is possible for two consecutive members of a series of eclipses to be seen from the same place. The total eclipse visible in England on Aug. 11, 1999, is indeed four Saroses later than that of June 29 of this year, but whereas this year's eclipse is in the early morning, the eclipse of Aug. 11, 1999, will be visible in England shortly before noon.

The circumstances connected with the recurrence of eclipses depend on several variables with different periods, and the apparent irregularity with which eclipses occur results from the incommensurability of the periods and differences in their relative importance. The most important period is that between successive new moons, which on the average is 29·5306 days. An eclipse of the sun would take place at every new moon if the orbital planes of the sun and moon coincided,[1] but as the inclination of the two planes is considerable—varying round 5°—it is only when new moon occurs near the line of intersection of the two planes, known as the line of nodes, that an eclipse can take place. On account of the motion of the plane of the moon's motion, the sun passes through the nodes at intervals of less than six months, the average time being 173·310 days, and this is the second important period in connexion with eclipses. Eclipses take place at intervals which are very nearly multiples of 29·5306 days and are approximately multiples of 173·310 days.

The maximum angular distance which the sun

[1] If this were the case, however, all central eclipses would take place within the tropics, and the only eclipses that could be seen from England would be extremely small partial eclipses at the new moons near midsummer.