Oceanic Isostasy in Relation to Geological Tectonic.[1]

By Sir JOSEPH LARMOR, F.R.S.

1. A CENTURY ago geodetic and gravitational universal surveys were mainly concerned with determining the effective (gravitational) ellipticity of the earth, after due allowance had been made for local anomalies, with especial view to the exact purposes of physical astronomy. One of the chief of these anomalies was exhibited by a remark of Airy, after scrutiny of the available data in his treatise (1830) on figure of the earth in the "Encyclopedia Metropolitana," that the observations show gravity to be abnormally in excess on island stations. It appeared, for example, that this cause might make the mass of the moon uncertain up to 2 per cent. A very refined explanation of this anomaly of island stations (which will be seen presently to be only partially effective) was offered by Sir George Stokes, from whom this last remark is quoted, in the course of a memoir,[2] fundamental for theoretical geodesy, in which he demonstrated that no outside survey could lead to any certain knowledge of the distribution of mass inside the earth, even in its outer crust, except as a matter of probability when backed up by geological knowledge.

It is explained there that the form of the sea-level must be locally depressed over a deep ocean, owing to defect of density; and in consequence on insular stations gravity at sea-level is measured abnormally nearer to the centre of the earth as a whole, so that from this cause its value is greater than that belonging to the mean spheroidal surface. In fact, the form of the ocean is an equipotential surface, including therein the potential of the centrifugal force of rotation in the familiar manner: but the part of the potential arising from the local water is abnormally small on account of its low density, and this defect must, in absence of local compensation, be made up by a greater potential of the earth as a whole, which demands depression of the local ocean surface towards the earth's centre.

The opposite result would arise from excess matter of an adjacent mountain or island peak: that would raise the ocean level in its vicinity and thereby indirectly diminish gravity, measured at sea-level as determined by levelling operations.

For example, at the centre of a circular oceanic basin or radius b and uniform depth h, its defect of potential would be with sufficient accuracy $\int \gamma\rho' h 2\pi r dr/r$, where ρ' is the defect of density of the water below that of the average terrestrial crust; thus it is $2\pi\gamma\rho' bh$, where γ is the constant of gravitation given by $\gamma E/a^2 = g$. Here $E = \frac{4}{3}\pi a^3 \rho$, ρ being $\frac{11}{2}$, is the mass of the earth of radius a. As the potential of the earth as a whole is $V = \gamma E/r$, this change of local potential, say δV_0, would be compensated by change of sea-level δh, where $\delta V_0/V = -\delta h/r$. Thus in the present case the fall of level relative to depth of ocean is given by the expression

$$-\frac{\delta h}{h} = \frac{a 2\pi\rho' b}{E/a} = \frac{3}{2}\frac{\rho'}{\rho}\frac{b}{a} = \frac{9}{22}\frac{b}{a},$$

while

$$\frac{\delta g}{g} = -2\frac{\delta h}{a}.$$

If the radius b of the oceanic basin is 50 miles this fall would be the fraction $\frac{9}{22} \cdot \frac{50}{4000}$ or $\frac{1}{200}$ of its depth; if the radius were larger it would increase in direct proportion until it is a considerable fraction of the earth's radius. A cup-shaped ocean could be similarly treated.

The steady sea-level would thus be depressed by $\frac{1}{10}$ of a mile owing to local causes, at the centre of a basin of 500 miles radius and 2 miles deep, in free communication with the other oceanic waters: and this approach to the earth's centre would involve increase of g measured at ocean level, given by $\delta g/g = -2\delta h/a$, or here $\delta g = 0{\cdot}05$ cm./sec.^{2}, where g is about 981, which is over one-third of the order of magnitude of the observed excesses at island stations.

But this explanation fails because there is a predominant offset. The vertical attraction of the local ocean regarded as an extensive flat slab of water is abnormally small by $2\pi\gamma\rho' h$, where $g = \gamma E/a^2$, that is by $g\rho' 2\pi a^2 h/E$ or $\frac{3}{2}\frac{\rho'}{\rho}\frac{h}{a}g$; thus this direct defect in g may be much the greater, being $\frac{1}{2}a/b$ times the indirect excess. There is however some effect in the other direction due to excess density of the local land, which is usually a substantial correction. This preponderance destroys and even reverses the Stokes explanation of the oceanic anomaly. Indeed closer examination shows that, as based by him,[3] rather confusedly as it seems, it depends on a potential equation used by Laplace which can, in limited manner, apply only to a locally infinitely thin spherical layer. The principle of depressed level became familiar, simple examples being worked out, *ab initio* and so correctly, by way of illustration in Chap. IV. of Col. A. R. Clarke's standard treatise on geodesy (1880), from the point of view however only of levelling operations, not of gravity.

But soon the discussion of the data of the Indian geodetic survey, by Archdeacon Pratt in India, revealed new features,[4] by showing strong residual defect of gravity on the Himalayas, such as could only be accounted for by a large defect of density underneath the mountains. Airy's idea that the mountains might be buoyed up by extensive roots floating in a denser magma, existing beneath a *thin* crust, could not of course now be maintained, at any rate in that form, in view of the high rigidity of the earth as a whole. But there was much to be said, on various counts, for a thinner and deeper viscid stratum, lying between the crustal material and the solid core, in which in the tendency towards equilibrium the pressure due to the weight of the crust must in course of ages have become

[1] Abstracted, with Sections 2 and 3 added, from *Proceedings of the Cambridge Philosophical Society*, Feb. 8, 1926.

[2] *Cambridge Transactions* (1849): reprinted in "Math. and Phys. Papers," vol. ii. Some idea of the great debt owed by the Indian and other gravitational surveys to the continuous amateur advice of Sir G. G. Stokes, spread over half a century of their development, may be gleaned from the collection of his "Scientific Correspondence" (Camb. Univ. Press), vol. ii. pp. 253-325.

[3] "Math. and Phys. Papers," vol. ii. p. 153. Stokes did not make any correction in this reprint in 1883; but Dr. Bowie states (*loc. cit. infra*) that there is no generally accepted explanation other than compensating excess of density beneath the ocean.

This analysis of Stokes in fact establishes as a general proposition that the effect of *distant* irregularities of surface mass consists of a direct vertical attraction, say g'', together with an indirect part due to change of level, equal to $-4g''$, thus countervailing four times: this influence, of wide range and presumably actually small, is superposed on the *local* effect here considered.

[4] In 1855-59: cf. A. R. Clarke, "Geodesy," pp. 96-98.

equalised laterally, at any rate partially, and the load upon it thus made uniform to that degree everywhere. It is implied that there are no local abnormalities of density in the core, which is reasonable as the core is probably metallic. This is the hypothesis of isostasy, propounded as a universal principle by Dutton and worked out systematically by Hayford and his colleagues of the American Survey, who found that it gave a fair account of the usually slighter anomalies (mainly of levelling) revealed in that great undertaking.[5]

Circumspection is, however, suggested in applying these ideas to the anomalies at oceanic stations; for the Stokes explanation already claimed to be an effective *vera causa*, without aid from compensation of density underneath. It happens that the subject is amenable in a general way to simple elucidation: and as the essential circumstances for submarine mountains and landscapes can perhaps be more directly estimated, it seems indeed to provide in some respects a closer test. On an ideal very narrow island-peak of negligible mass, in a wide ocean of uniform depth, with adjustment as a whole to general isostasy by denser horizontal strata underneath, there would be but slight resultant abnormality of the local part of the attraction. For the totality of the strata could almost be regarded as an extensive thin flat sheet, while local defect of potential on which change of sea-level depends would be still more closely compensated by the extra mass below.[6] Hence, in contrast to the Stokes uncompensated case above, under isostatic conditions gravity and level ought both to be regular over a wide ocean of nearly uniform depth with strata nearly horizontal underneath.

2. The distribution of gravity over an oceanic surface, beneath which local compensations of terrestrial density are taken to be complete, may thus be envisaged, perhaps most simply, by drawing a widely extended arbitrary horizontal boundary beneath the water, and marking out all above it up to the level surface as ocean separately compensated beneath, the law of depth of the compensation being for that hypothetical layer of the density of water unimportant. There will then remain the effect of the surplus of density, over the oceanic water, of the solid parts situated above this arbitrary flat boundary; and it is from this reduced submarine mountain-landscape alone, together with emergent peaks with density undiminished, and the nature of its compensation, that the amount of the actual local excess of gravity is to be estimated on the hypothesis of isostasy, the circumstances thus being analogous to those of a range like the Himalayas, but modified, as all the observations now belong to the same level near the tops of the submarine mountains instead of the bases. The nature of the compensation, in the deep-seated material, of this effective local excess load, would thus permit of being judged by itself; in particular, for steep submarine island peaks it is almost negligible, whatever varying distribution in depth be assigned to it, provided only it extends deep down, say towards the order of 10^2 kilometres.

The long-recognised excess of gravity at island stations was thus really evidence quite as forcible, and also as direct, as the subsequent records of Himalayan surveys, indicating that the defect of density of the masses of water is actually compensated, even over wide uniform oceans, at any rate to a very considerable degree, by excess of density below.[7] The systematic discussion of the level and gravity surveys of America, primarily by Hayford, has enlarged and forced into prominence the same very striking and surely fundamental type of conclusion, as extended even to the usually smaller and less abrupt anomalies there revealed.

The evidence, then, is on all sides remarkably strong, that with increase of depth the terrestrial material gradually becomes softer, so to say, possibly owing mainly to rise of temperature, down to a limit which perhaps at an outside estimate may approach 10^2 kilometres: that below some such depth the mass of the earth presents again a perfectly solid, though doubtless elastically deformable, foundation on which the softer strata directly above have flowed gradually in the course of ages towards an equilibrium nearly hydrostatic, depending in detail, however, on the distribution and range in depth of the softness, in a way that is scarcely much amenable to scrutiny. To effect such adaptation, the displacement of deep-seated material need be only over slight distances, unless the yielding layer is thin. An unyielding foundation underneath is essential to any approach to local isostasy; the earth as a whole must be solid, as it is known to be for dynamical reasons. As regards the relatively shallow upper terrestrial layer which thus becomes viscous with depth, in a way not necessarily uniform nor to the same depth everywhere, the question of rupture or damping of transmission of internal earthquake tremors in crossing these softer layers arises, and is probably ripe for discussion; such a stratum may of course be even completely yielding for slow secular stress while thoroughly elastic for the rapid alternations in seismic oscillations. It is to be remarked, however, that as a result of theory superficial travelling waves, at any rate on uniform elastic material, could scarcely arise from other than a superficial cataclysm, secondary it may be, so that purely superficial seismic undulations would have to come from sources located within their own quite small range of depth. But the velocity would change (dispersively) with wave-length, and this conclusion may be modified, as Prof. Love pointed out, if the elastic quality or density, instead of being uniform, changes notably within the depth of a wave-length.

Why distinct settlement of the strata towards isostasy such as is thus variously confirmed should be necessary at all, affords direct scope for fundamental tectonic speculation, of an interest quite apart from geological detail. Is this abnormally small density beneath mountain ranges due to higher temperature or to lighter material? How could such locally varying temperatures have become established over a consolidating earth? If the height of the mountains is determined largely by the defect of density beneath, they must to that degree have been pushed up hydrostatically from below rather than elevated by lateral stresses; yet folding of the mountain strata is con-

[5] Cf. the chapter in H. Jeffreys' recent treatise "The Earth."

[6] In the case illustrated above, with radius of ocean about 500 miles and depth of compensation 100 miles, about 10 per cent. of the anomaly both of attraction and of potential would remain after compensation of the ocean.

[7] For recent special estimates see a note by W. Bowie, *Proc. Washington Acad.*, Dec. 1925.

spicuous. Subsidence towards isostasy might perhaps induce folding to some degree. If the depression of the Pacific Ocean is thus determined in the main hydrostatically, is there not less room for the cosmic theory that it may represent the cavity from which the moon was originally shed away?

3. *Postscript.*—One observes that these and cognate questions, insistent and fascinating, form the subject-matter of Prof. Joly's recent path-breaking book, "The Surface History of the Earth," which invokes steady evolution of heat by radioactivity of the rocks, interacting with isostatic influences, as the cause of periodic outbursts of surface activity which have fashioned the existing features. There are to be compared the views developed in H. Jeffreys' recent comprehensive treatise, "The Earth." For a condensed account over an extensive range *cf.* "A Symposium on Earthquakes," by F. A. Tondorf, N. M. Heck, W. Bowie, A. L. Day in *Journal Washington Academy*, May 4, 1926, pp. 233-254 (also more recently G. R. Putnam). In a less special way, such questions have been prominent since the treatise of E. Suess on the earth's surface features. There is also the problem of the time-scale of development, projecting into vast æons of the past, yet with clues arising mainly from the fossil traces of the succession of forms of life.

A few special remarks may be significant here.

It appears that the lag in compensation of accumulating great depths of sediment is but small, compared at any rate with the time of accumulation, for the compensation is always well advanced.

Tidal pulls on these adjustable surface-sheets would on Newtonian principles be differential, and so extremely slight. Thus even the extreme case of an elastic earth surrounded by an ocean of molten lava of the order of 10^2 kilometres in depth, in which continents would be analogous to ice-sheets and mountains to icebergs, is not unthinkable dynamically, however it be thermally; though the existence of the actual oceanic tides would demand a rigid and deep crustal layer.

But even if the lagging tidal pull were large enough, it could only cause a westward drift of the fluid surface material around the earth as a whole, not of continents and mountain ranges floating thereon. For the principle of Archimedes asserts itself; as regards the uniform field of force the floating mass can be replaced by the magma which it displaces, up to the level surface; thus it is the same as if the tidal forces acted on a uniform sheet of magma without surface excrescences and no differential drift could arise—except in so far as a uniform drift may be obstructed or deflected locally by the more solid roots of the floating continents that are carried along with it.

The earliest table-lands, of primitive rock, must have been pushed or floated up, and to great heights; it would appear from the literature that their subsequent denudation by aereal influences accumulated stratified deposits along the coasts of the oceanic hollows, which gradually sank into the magma by their own extra weights, perhaps most in the middle so as to curl over by the lateral pressure,—themselves sinking down while the adjacent denuded high land is floated up, until by accumulation combined with sinking, and helped by effusions from below, they attained to considerable slopes and great thicknesses, even five miles or more, that then somehow they were pushed up again bodily, yielding after repetitions of such processes folded mountain-ranges of stratified rock such as geologists know, the primitive elevations having passed largely out of sight. At any rate nothing more plausible seems to have been hitherto thought of.

The Golden Eagle.

By SETON GORDON.

"Thrice the age of a dog the age of a horse,
Thrice the age of a horse the age of a man,
Thrice the age of a man the age of a stag,
Thrice the age of a stag the age of an eagle,
Thrice the age of an eagle the age of an oak tree."
—*Old Gaelic saying.*

THE golden eagle is the most magnificent bird of the Scottish highlands. Up to the middle of last century, the erne or white tailed eagle shared the cliffs of the western seaboard and islands with the golden eagle, but the erne is now extinct, although so late as the middle of the last century almost every headland of the Isle of Skye had a pair of these fine birds nesting upon it.

The flight of the golden eagle has inspired many poets, and from the earliest times it has been looked upon as lord of the air. In the Book of Proverbs we read:

"The way of an eagle in the air,
The way of a serpent upon a rock,
The way of a ship in the midst of the sea,
The way of a man with a maid."

These, says the writer, are the four things too difficult to understand. Keats wrote in 1818 "Eagles may seem to sleep wing-wide upon the air," and how descriptive are Wordsworth's lines "Faint sound of eagle melting into blue." Scott writes that the eagle from her rocky perch on Ben Venue "spreads her dark sails to the wind."

The eagle is the royal bird of Greek mythology; an eagle of gold was the standard of the Romans. At least three countries have the eagle as their emblem: Assyria, Persia, and Rome. It is, or was, the national arms of France, Germany, Russia, Italy, Austria, and Poland.

At the present day the golden eagle is confined to the central and western highlands of Scotland. In the more accessible districts it is terribly harried by egg collectors, and here very few eyries escape. It is unfortunate that the golden eagle, almost alone among Scottish birds, should never lay a second time in a season, even if its first clutch of eggs be taken when quite fresh, but, despite the egg collector's zeal and the keeper's gun, I do not think the eagle is on the decrease, except here and there. Each pair of eagles has two, sometimes three, eyries. The same eyrie is seldom used two years in succession, because the eaglets remain long (about eleven weeks) in the nest and from