The Cooling of the Earth.[1]

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THERE are strong reasons for believing that the earth was wholly fluid at an early stage of its history. My present topic is the manner in which such a fluid earth would solidify and afterwards cool to its present state.

The best known account of the solidification of the earth is that given by Kelvin. Most of the rocks that constitute the crust contract when they solidify. According to Kelvin, then, the first stage in solidification was the formation of a thin solid shell on the outside. But this shell was denser than the liquid interior, and therefore was unstable when floating on it. The shell therefore broke up and the pieces sank. A new shell then formed on the outside and the process was repeated, until the liquid was replaced by a sort of honeycomb; the cells filled with magma might become the seats of later vulcanism.

This account requires to be modified to allow for two facts. Pressure raises the melting point, and also the temperature of a rock material. Also the earth is not composed of a single material, but of several with widely differing densities and melting points. We wish to know how far these facts will modify the mechanism described by Kelvin.

Taking the former effect first, let us consider what would be the actual course of events in the solidification of an earth composed of a single rock material and cooling by radiation from the surface. Matter cooling at the surface would thereby contract and become denser than that underneath, and therefore would sink, its place being taken by other matter from below. Thus the whole would be continually stirred up. But when the matter descends it enters regions of greater pressure, and consequently is heated. The rise of temperature is estimated by L. H. Adams, of the Geophysical Laboratory at Washington, as somewhat less than 1° C. per kilometre of depth. So long as the earth remained fluid, then, the temperature in it would decrease downwards at this rate.

Now pressure raises the melting point of rocks to the extent of 3° C. for the pressure due to the weight of a kilometre of rock, much more than the effect of pressure on the actual temperature of a specimen of fluid. Thus, while the earth was fluid, the difference between the temperature of the fluid at any depth, and the melting point at the same depth, was greatest at the surface and least at the bottom. The temperature therefore reached the melting point first at the bottom, and solidification started there (at the centre of the earth, that is, since we are considering an earth of uniform composition). Cooling and agitation continued at higher levels, and the solid layer gradually thickened until it reached the surface. Thus the honeycomb structure would not be produced.

Coming now to the differences of material within the earth, we can exhibit the principal constituents in the accompanying table.

The materials are here arranged in order of increasing density. They probably form fairly continuous layers in the earth, the denser at the greater depths. The first four are frequent at the surface, but the Femi is known only in deep-seated intrusions, and the nickel-iron alloy is only believed to occur deep down in the earth because some such material is necessary to account for the high mean density of this planet. The names Sal, Sima, Femi, and Nife are due to Suess; each combines in abbreviated form the names of two characteristic constituents of the material.

Layer.	Typical Rock.	Specific Gravity.	Melting Point.	Chief Constituents.
Air	..	10^{-3}	−200°	..
Water	..	1	0°	..
Sal	Granite	2·66-2·72	600°-1000°	SiO_2, Al_2O_3
Sima	Diabase, Basalt	3·0	1200°	SiO_2, FeO, CaO, MgO
Femi	Peridotite	3·3	1400°	SiO_2, MgO (in excess), Fe_2O_3, Fe_3O_4
Nife	Nickel-Iron	8·2	?	Ni, Fe

One of the first events in the formation of the earth was the settlement of the Nife to the centre. We know from the theory of the figure of the earth that its boundary is about 1400 km. down, so deep that it can have had no important effect in the evolution of the upper layers.

If the water of the ocean was originally in the atmosphere, in the form of steam, the pressure of its vapour at the earth's surface would be of the order of 300 atmospheres. It is known by experiment that at such a pressure melted granite and water mix freely. It is therefore probable that nearly all the water was initially within the crust, dissolved in the rock magmas. It is more doubtful whether the principal rock types could mix with each other when fluid. Experimental evidence shows that they could in some conditions, but their present mode of occurrence indicates that at some stage in the development a certain amount of separation took place. It is probable that a gradual solidification of the denser and less fusible rocks led to a concentration of the water in the granitic layer, and that much of it was extruded from the last itself when it solidified.

The solidification would start either with the Nife or at the bottom of the peridotitic layer, and would extend upwards as was described for a homogeneous earth. The liquid above would remain in adiabatic equilibrium until no further femic material remained liquid. The temperature anywhere in the solid portion would evidently be the melting point at the depth considered. Hence the rate of upward transmission of heat by conduction in the solid layer is calculable: it would be about $1{\cdot}5 \times 10^{-7}$ cal./cm.^{2} sec. But the rate of loss of heat from the outside by radiation would be more like 1 cal./cm.^{2} sec. Thus the liquid layer would go on cooling by radiation almost as if no heat was being conducted into it from below, until the temperature at its base reached the melting point of Sima, when the rocks of this type began to solidify. When the Sima was solid, further cooling led to the solidification of the Sal layer; at some stage of the process the water separated and an ocean formed.

As soon as the earth was solid at the outer surface, the great excess of the heat lost by radiation over that

[1] Based on a lecture delivered before the London Mathematcial Society on April 23.

conducted from the interior would ensure that the temperature of the surface rapidly fell until the loss of heat from the surface nearly balanced that received from the sun. The sun was at that time probably radiating about as intensely as at present, so that the equilibrium temperature of the surface would also be nearly the present temperature. Thus the primitive solid earth would have much the same surface temperature as now, but a temperature equal to the melting point of peridotite was reached at, or a little below, the top of the femic layer, perhaps 40 kilometres down. Cooling to this stage probably took some thousands of years from the formation of the earth.

The cooling of the earth, from the stage just described, down to its present condition, was a much slower process. So long as the outer layer was liquid any cooling on the outside would lead to turbulence, and therefore the whole of the liquid layer would cool equally fast. But when the earth had become solid, conduction became the only agency available to redistribute its heat, and conduction is very slow. The beginnings of a quantitative discussion of the point were made by Fourier, who showed that if we have a uniform rod, infinite in both directions, and with initial temperature $f(x)$ at distance x from a fixed point of the rod, the temperature at time t is given by

$$V=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} f(x+2qh\sqrt{t})e^{-q^2}dq,$$

where h^2 is the thermometric conductivity. In the earth the region is not infinite in either direction; but it is found to be a good enough approximation to suppose it infinite downwards and to treat the flow of heat as one-dimensional, since the depth where cooling is considerable is a small fraction of the radius. The other boundary condition is that the temperature at the surface is maintained constant. Lord Kelvin made the first important contribution to this problem, as to that of the method of solidification, though in this case also his discussion has needed much revision to bring it into accordance with later experimental knowledge. He supposed the temperature at all depths, 0 to infinity, to be uniform and equal to S, the melting point. The difficulty of the constant surface temperature was met by replacing the earth by a solid infinite in both directions, the temperature being antisymmetrical with regard to the surface. Thus subsequent conduction would keep the surface temperature constant, and Fourier's solution could be applied as it stood.

Kelvin's solution was

$$V=S\ \mathrm{Erf}\ \frac{x}{2h\sqrt{t}},$$

the symbols having the meanings already given. Erf is the Error function, defined by

$$\mathrm{Erf}\ \lambda=\frac{2}{\sqrt{\pi}}\int_0^{\lambda} e^{-q^2}dq.$$

Differentiating this and then putting x zero, he found

$$\left(\frac{\partial V}{\partial x}\right)_{x=0}=\frac{S}{h\sqrt{(\pi t)}}.$$

This equation was used by Kelvin to estimate the age of the earth. The left side is the rate of increase of temperature downwards at the present time, determined from observations in mines and borings. With modern data this is $32^{\circ}\times 10^{-5}$/cm., and with S equal to 1400° and h as 0·084 c.g.s., the equation makes t equal to 27 million years.

The increase of temperature downwards would alter this equation slightly: its effect is to introduce into the temperature a term of the form mx, which does not change with the time. Allowing for this we find that the estimate of t needs to be increased to 33 million years.

These estimates, however, were found to need drastic alteration soon after the discovery of radium. Radioactive matter was discovered to be universally present in rocks, to such an extent that in an average granite it is generating $10{\cdot}1\times 10^{-13}$ cal. per c.c. per second; the amount for a basalt is $2{\cdot}7\times 10^{-13}$ cal. per c.c. per second. These amounts appear small, but then so is the loss of heat from the surface. The latter is about $1{\cdot}6\times 10^{-6}$ cal. per square centimetre per second. Thus a layer of average granite 16 kilometres thick would account for all the heat leaking out of the earth. This remarkable result was obtained by the present Lord Rayleigh. That it demanded careful reinvestigation of the theory of the cooling of the earth was obvious; and some writers went so far as to deny that the earth is cooling at all. The situation, however, was never so serious as this. Either the total radioactivity is less than that of 16 km. of granite, or it is greater. If it is less, the earth is cooling to some extent, though less than the simple Kelvin theory indicates; if it is greater, it is impossible to explain why the amount of heat being conducted out of the earth is as small as it is, for the heat generated must be going somewhere.

A way out of the impasse was found by Dr. Arthur Holmes, in a series of papers in the *Geological Magazine* for 1915 and 1916, which have not yet attracted from physical writers the attention they deserve. Taking the extreme case of no cooling, so that the temperature within the earth is now everywhere steady, he worked out the depths of rock of various types needed to give the observed surface temperature gradient, and hence the temperatures at various depths. He found that if the radioactive layer was average granitic rock, the temperature within the crust could nowhere exceed that at the surface by more than 300°. Such a temperature is quite inadequate to explain the occurrence of volcanoes and igneous intrusions within the continents, and points definitely to another source of heat, which it is natural to refer to the primitive store.

Holmes therefore assumed a cooling earth, with radioactivity falling off exponentially with the depth; the mathematical solution of this problem had been given by Ingersoll and Zobel, but not applied to the actual conditions of the earth. If the rate of generation of heat per unit volume is Ae^{-ax}, it was found that

$$\left(\frac{\partial V}{\partial x}\right)_0=m+\frac{S}{h\sqrt{(\pi t)}}+\frac{A}{ak}\left(1-\frac{1}{ah\sqrt{(\pi t)}}\right).$$

This equation contains now two unknowns, t and a. Thus it can no longer be used for finding t; but that does not matter, because radioactivity gives us an independent determination of t. The disintegration of uranium produces lead at a known rate, and hence the analysis of a uraniferous rock makes it possible to estimate the time elapsed since that rock crystallised.

By this method the age of the oldest rocks known has been determined as 1400 million years, and it is probable that the whole age of the earth is about 1600 million years. With this extra datum we can find a. It turns out that radioactivity at a depth of 13 km. must be $1/e$ (that is, 0·37) of what it is at the surface, that at 26 km. $1/e^2$, and so on. On this basis it is found that the differences between the present temperatures at various depths and the melting points of peridotite at the same depths are as follows:

Depth (km.):	0	37	74	111	148	185	222	259	296	333	370	444	518	592
Temperature differences (degrees C.):	1400	940?	830	710	600	510	420	340	280	210	170	95	50	25

Below 600 km. or so the cooling is inappreciable. The cooling at depths of 200 to 300 km. is not so great as to forbid occasional softening of the more fusible constituents of the Femi, so that the existence of vulcanism is consistent with these estimates.

I have performed the corresponding calculations for another hypothesis differing as far as possible from that of Holmes; namely, I supposed the radioactivity uniform down to a finite depth and zero below that depth. The effect of the change is not great: the cooling at all depths is increased by about 16 per cent.

The above calculations are based on numerical data differing somewhat from those used by Holmes, and also from those used by myself in previous work. Previously I used as the primitive surface temperature the melting point of basalt, 1200°, seeing that it was the deep-seated rocks whose initial temperatures would have the greatest influence in determining present temperatures. But L. H. Adams,[2] in a recent rediscussion of the whole matter, has pointed out that I did not go far enough, and adopts 1400° as his standard melting point; this datum has been used above. At the same time he has made an allowance for the difference between the conductivities of rocks at different depths. The effect is to increase the amount of radioactive material and reduce the cooling, but the general trend of the results is not violently changed.

[2] Jour. Wash. Acad. Sci., 1924.

The Royal Academy Exhibition.

IN a little book on "The Revolutions of Civilisation," with abundant illustrations of the arts of many ages, Sir Flinders Petrie has sketched out a sequence of rise and decline of civilisations in eight periods from the dawn of history, six of them between 6000 B.C. and A.D. 2000. It is through the arts that the sequence is manifest: the several arts keep an order of precedence, they reach in turn a maximum of development; and in turn decay. In each period sculpture is the first of the arts to reach its maximum phase, followed by pictorial arts and then in turn by literature, mechanics, and finally by wealth. So also, in each period, the first signs of decay are manifest in sculpture; the decay of pictorial arts comes next. Medieval civilisation developed its maximum phase of sculpture in the thirteenth century, of painting at the end of the fourteenth, of literature at the end of the fifteenth; we are now in the maximum phase of mechanics, and all we have in prospect before our period goes out and the ninth becomes dominant is a maximum of wealth.

Suppose a visitor properly imbued with these ideas of revolutionary civilisation should find himself among the Royal Academy pictures of 1925, with room to see, and leisure to think about, the fourteen hundred items of the exhibition, what impression would he get? What would he think of sculpture which already showed signs of decadence four centuries ago? It is represented by reliefs, as 1273, *The Late Bishop of Hereford* for his Cathedral, by Allan Wyon, recumbent statues of *Lord Kitchener* in marble (1381) for St. Paul's, W. Reid Dick, and *The Late Bishop of Coventry* in bronze for his Cathedral (1377), by Sir Hamo Thornycroft, R.A., many figures, busts and statuettes, and some really convoluted animals, *Wild Swans* (1222), *Eagle, Lynx, and Hare* (1223), by the Danish sculptor Holger Wederkinch. Is a recumbent statue the imitation of a bygone habit of centuries ago or a step in the progress of the realisation of an art which also strives to represent action, as in a bronze *Atalanta* (1414) by Sir Bertram Mackennal? After he had assigned the position of sculpture, between the failing light of the eighth period and the dawning of the ninth (making what allowance is necessary for "copying," with which Sir Flinders Petrie declines to concern himself), what would he think when confronted with 477, *Sir Donald MacAlister of Tarbet*, as portrayed by Maurice Greiffenhagen, R.A., or 79, *A Street Accident*, by Glyn W. Philpot, R.A., or 340, *The Soul's Journey*, according to Mrs. A. L. Swynnerton, A.? How would he relate them to the golden age before the cinquecento? What, anyway, could the student of civilisation have said if 160, *Man Versus Beast* (*Paris*), Sir William Orpen, R.A., happened to have been unearthed from an Egyptian tomb instead of being exhibited as a novelty in a London gallery?

It is a well-arranged exhibition: the oil paintings, which number only 631, are hung within comfortable view, mostly in not more than double rows. These are supplemented by 407 water colours, miniatures, drawings, engravings or etchings in the South Rooms, 174 architectural drawings and 207 sculptures.

One gets the impression of alternations of portrait and landscape with very few historical or subject pictures, more uniformity of excellence and fewer striking exceptions than usual. There are, once more, a number of examples of brilliance of illumination by La Thangue obtained by juxtaposition of light and shade: 42, *Amalfi Vines*; 84, *A Provençal Flock*; 141, *The Thorn*; and 175, *The Trout*. There are some efforts of a similar character not nearly so successful: 305, *Jack, Jill, and Peter*, Dorothea Sharp; 407, *A March Morning*, Harry Fidler; and better than these 537, *Eucalyptus Avenue*, Mary H. Carlisle. There are also striking examples of moonlight brilliance by the juxtaposition of iridescent colours; 14, *Silver Moonlight*, and 129, *The Ebbing Tide*, Julius Olsson, R.A.

For the spectator, whose days belong to science and to whom the technique of art is a mystery, the landscapes naturally afford more food for reflection than the portraits; and the comparative uniformity easily leads to thinking about the colour schemes of Nature, as expressed by different artists. There is a whole gamut of variation between the blue middle distance and