Why Wireless Electric Rays can bend round the Earth.[1]

By Sir JOSEPH LARMOR, F.R.S.

MYSTERY has remained attached to the transmission of free electric rays a long way round the protuberant curvature of the earth, which has recently developed into the greatest sudden practical evolution in signalling since the telephone. The difficulty was already emphasised by the late Lord Rayleigh, as soon as the first signs of transmission across the Atlantic had been detected by the Marconi operators. The effect has been sometimes supposed to be accounted for by a hypothesis that the rays are turned downward by an upper conducting layer in the atmosphere. But conduction, as usually understood, involves dissipation, and thus loss of energy of the rays by absorption; so that a train of radiation travelling along a layer sufficiently conducting to bend the rays could not go far. In fact, by a well-known dynamical principle, if the absorption is small of the first order, the resulting increase of velocity of the train is small of the second order and so of no account for bending rays in a varying field.

There seems, however, to be a sufficient and rather striking cause available for transmission of long waves horizontally round the earth at great altitudes, though the rays travelling at lower heights would be gradually quenched. For the synchronous oscillations of free ions of small mass go on undisturbed by collisions in very high regions of the atmosphere, and have time owing to the long period of the exciting waves to get up high speed and so become important under the influence of the field; much as the electrons in a triode valve have time during the electric oscillation affecting them, or rather in this case during their passage, to get deflected on to the grid. In an extremely tenuous atmosphere, this free ionic oscillation would be almost the sole conceivable deflecting agency. On the other hand, the absorption that would accompany ionic oscillations in ordinary conduction arises from the shortness of the free path, which leads to dissipation, through collisions with the surrounding molecules, of part of the extra energy acquired in the accelerated motion of the ions and thus abstracted from the electric field. When, as above, the period and amplitude of the oscillatory motions of the ions are much shorter than the time and length of their free path, so that the great majority of the oscillations take place without interference from collisions, this loss of energy will be very much reduced. A rough illustration of this principle is doubtless afforded, at the other extreme, by the remarkable penetration of the very short X-rays into light metals. It is not, of course, that conductance tends to become perfect, for then no waves could travel: the influence must be turned on to the dielectric modulus of the medium.

This cause must operate also for long waves travelling in the extreme limits of the atmosphere where the free path is great. The density aloft may be extremely minute: the long swing of the ions can bend the waves. The region in which the aurora borealis is displayed extends up to a height of fifty miles or more, and there the free atomic path for air would be about two centimetres. For radiation of wave-length of a kilometre, with oscillating field of amplitude one volt per metre and so transmitting $\frac{4}{3} \cdot 10^{-7}$ watts per square cm., the amplitude of the oscillations of free electrons would be about five centimetres, and for heavier ions it would be smaller inversely as their mass; while the time of oscillation for this length of wave is only about a twentieth of the time of the free path, at this altitude, for a molecule of the air, or a quarter of the free time for a hydrogen ion. For an electron moving with the speed of thermal equilibrium this time of oscillation would come to ten times the time of its free path at this height; this would be entirely excessive and the electron would be an agent of strong dissipation, were it not a reminder that the electron passes through the atom each time with but slight derangement of its free motion.

The long free path would in itself tend to augment the degree of ionisation. There must, of course, be as many positive ions present as negative; but though both influence the velocity of propagation of the waves in the same direction, the effect of the massive ones is negligible. It can readily be shown that the result is to reduce the effective dielectric modulus of the medium, the value of which is close to unity, by $N_0 e^2 \lambda^2 / 2\pi m$, where N_0 is the numerical density of the ions of charge e and small inertia m. The amplitude of free oscillation of each ion would be $e\lambda^2/4\pi^2 mc^2 \cdot F_0$ in a field of amplitude F_0. The velocity of propagation v is increased relatively by the first of these amounts, which is for the same stratum proportional to the square of the wave length λ. The curvature of the horizontal rays in any stratum is equal to $d \log v/dh$, the rate of relative increase of the velocity upwards, which is here $\frac{e^2\lambda^2}{2\pi m}\frac{dN_0}{dh}$. The stratum of transmission to great distances is the one for which this is exactly the same as the curvature of the earth, namely, $\frac{1}{2}\pi \cdot 10^{-9}$. For electrons this gives dN_0/dh equal to 10^{-5}, for hydrogen ions 1700 times this; thus if this adaptation of curvature applies to a sheaf of rays extending over the height of as much as a kilometre, N_0 need not mount to more than one electron per cubic cm. for this wave-length of one kilometre, whereas the number of the atoms would still be as much as 10^{14} per cubic cm. at the height of fifty miles, which gives a sufficient free path for material ions. For hydrogen ions, N_0 would have to be increased to $2 \cdot 10^3$ per cubic cm., while the amplitude of oscillation would be reduced 1700 times so that the effective stratum need not be so high; if free electrons also are present in numbers, in thermal equilibrium, they would introduce dissipation of the rays passing at levels considerably higher than this rough illustrative estimate of fifty miles,—were it not that their motion, more potent on account of their smaller mass, is so slightly deranged by passing through an atom as to compensate for the much greater number of such encounters.

These considerations remain valid in a general way when the strata of equal ionisation are not exactly horizontal. If any cause, such as the influence of sunlight on the ionisation, alters the height of this

[1] Abstract of a paper read on October 27 at the Cambridge Philosophical Society.

effective stratum too suddenly, the rays will be bent away upward or downward, and scattered, at the place of dislocation, and may not be able to get adjusted into the new stratum of transmission. If, however, the stratum is thick, the dislocation will be incomplete, extending over only part of it. The stratum may be expected to be of varying height, some function of the local solar time; but it is too high for any merely meteorological derangement. All rays emitted not nearly horizontally are lost: a good local earth would assist the antennas here. We can think of the constituent beam of rays connecting the transmitter with a receiver anywhere along the path: it travels most of the way, without loss except by spreading sideways, in the effective stratum, in which all such beams unite to form a nearly horizontal band of rays, the rough analogue of an optical caustic, in the almost vacuous region above. Each receiver collects from an area around it of the order of the square of the wave-length: it thus appears from numerical estimate that the amount of energy available need not be at all inadequate to account for the now familiar features of free electric transmission round the earth, even to the antipodes and beyond.

Obituary.

DR. J. E. CAMPBELL, F.R.S.

THE sudden death, at sixty-two years of age, of John Edward Campbell has come as a grievous shock to many. It removes from among us, in the height of his powers, a pure mathematician of strong individuality, and of conspicuous achievement in departments of research where few, in the British Isles at any rate, could bear him company. It deprives the small fraternity of mathematical lecturers in Oxford of a recognised leader, both revered and beloved. It means the loss to his College of a tutor no less devoted and successful than distinguished, who also served it in administration as acting head during the last years of its great Principal, Dr. Boyd. It leaves certain causes of philanthropic and religious work in Oxford sadly the worse off for working friends.

Campbell was the son, born at Lisburn, Co. Antrim, in 1862, of Dr. John Campbell. After going through the Belfast College of the Queen's University with distinction he came to Oxford, rather older than most undergraduates, as a scholar of Hertford. Later he became fellow and tutor of the same College, after gaining all the ordinary academical honours. He was also for a good many years lecturer of University College. He became a fellow of the Royal Society in 1905. In the years 1920–22 he was president of the London Mathematical Society; while the University of Belfast made him an honorary doctor of science.

Geometry was the strong point of Oxford mathematics in Campbell's young days, and it was natural for him to choose his line of research with a certain geometrical predilection, but he never became a devotee of the pure geometry of Chasles like others who surrounded him. Prettinesses did not much appeal to him: he wanted to plough broad acres. Looking afield for them, he began the study of the theories of continuous groups and of contact transformations which that master-geometer Sophus Lie had exploited, and with Sophus Lie he stayed long. Most of his earlier writings were contributions to Lie's theories, and presently he wrote a comprehensive book upon them, the first one by a British author, and far from a slavish reproduction of Lie's ideas. The differential geometry of surfaces has for long been another subject of his study and productiveness. More recently he has plunged into the time-space of Einstein in hopes of finding firm ground, as witness his presidential address to the London Mathematical Society.

There was a wonderful charm about Campbell's personality. Transparent rectitude, playful humour, the courage of his opinions, all shone from him to those who knew him. Affectation was as alien to his nature as idleness, and he could not be ungenial. In fact, he was the sort of Irishman whom it is a joy to run against. Some, hearing him for the first time, may have fancied that he had brought his Scotch name and his quaint accent unfamiliar to southern ears from somewhere to the east rather than the south-west of Campbeltown, and perhaps a comparatively remote Scottish ancestry may have had to do with the sturdiness of his principle and the forcefulness of his determination. Patient he was at his work, tolerant of stupidity, kindly in his joking, but there was a fire underneath which showed itself when what he thought wrong was in question. In the middle of 1914 he was a heated Ulster patriot. August came, and the lesser patriotism was submerged by a flood of the greater. A dearly loved son was an early sacrifice. During the War there was nothing else to him that mattered. He lived to serve those who fought for the right.

The War over, Campbell returned to his mathematics and to his tutorial work with young men. Of the latter, another generation or two have learned to be thankful for his precept and example. Of the former, a new book from his pen—one on differential geometry—will shortly speak.

WE much regret to record the death on October 4, at the age of seventy-seven, of Dr. Joseph Moeller, emeritus professor of pharmacognosy in the University of Vienna, and we are indebted to the *Chemiker Zeitung* for the following details of his life and work. Dr. Moeller studied in Vienna and was appointed an assistant in the pharmacological department in 1871. After the publication of his work, "Vergleichende Anatomie des Holzes," he lectured on natural products in the technical high-school at Vienna, and he was awarded the *venia legendi* for pharmacology of the University. In 1886 he was appointed professor of pharmacognosy at Innsbruck, and in 1893 he left that University for Graz. Later he returned to the Medical Faculty of Vienna, where an independent Institute of Pharmacognosy was built for him. Among other publications Dr. Moeller was the author of "Mikroskopie der Nahrungs- und Genussmittel," which has been translated into several languages; "Anatomie der Baumrinde"; and, with Ewald Geissler, of the "Real-Encyklopädie der gesamten Pharmacie."