

into gold in the mercury lamp of Jaenicke, when burning overcharged. I resolved to return to my proposed experiment, and I did not intend to study mercury but lead. In accordance with this original scheme, I have now carried out some experiments. Peculiar difficulties arise, because lead fuses at 327°C ., and also because lead, solidified in a quartz vessel, breaks the vessel if heated again. Notwithstanding this, we succeeded in constructing a satisfactory lead quartz lamp (Fig. 1). The two legs A and B end in two narrow tubes, in which two steel electrodes are cemented with sealing wax. Around these cemented ends two little copper water coolers G and H are inserted, preventing fusion of the sealing wax. Vessel C is the store vessel, into which the pure liquid lead is poured, after which the open end is sealed off. An electric furnace around C keeps the lead liquid. Tube D, which is bent, contains a capillary F and ends in a cock K. The free end of the cock is connected with the mercury diffusion pump.

When a high vacuum is reached, the vessel C containing the lead is heated to redness with the Bunsen burner, with the result that all oxide is dissociated, and that

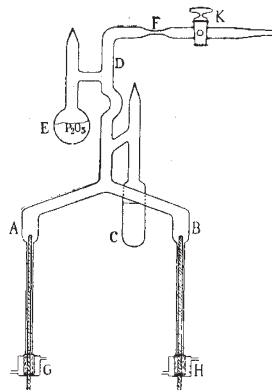


FIG. 1.

the metal, freed from all gases, has a surface as smooth as pure mercury. After closing the cock K and breaking the connexion with the pump, the apparatus is inclined, and the liquid lead is run into the two legs of the lamp. Now the lead is once more heated to redness in the two legs to drive out the gases from the steel electrodes. Many gas bubbles escape now and the apparatus is evacuated once more. After that the lamp is ready, and can be used in the same way as the mercury lamp, with this difference only, that removable radiating coolers cannot be placed around the legs A and B before the lamp is working. At the end, the liquid lead is again brought into the vessel C, which is continuously heated by the little electric furnace at 350° .

Because bismuth is also an element which is to be considered here, and the system lead and bismuth has a eutectic point at about 120° , it was of interest also to try this eutectic composition. We found that this alloy can be used in exactly the same way.

It is clear that the investigation of these lamps is not only of use to show whether transformations of elements can be realised, but also to study the arc spectrum. The lamp here described, like the vacuum mercury lamp, burns at relative low voltage; we are constructing another lead lamp which, like Jaenicke's lamp, burns at higher voltage. We hope to be able to communicate some results shortly.

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An Approximation to the Probability Integral.

THE probability integral

$$\Theta(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2} dt$$

can be represented by the simple approximation

$$\frac{6}{\sqrt{\pi}} \cdot \frac{t}{3+t^2} = 3 \cdot 385 \frac{t}{3+t^2} = 1 \frac{1}{8} \frac{t}{1+\frac{1}{3}t^2}$$

over such a range ($t < \sqrt{3}$) as to make the approximation useful to the statistician and others whose problems do not require great accuracy.

Since

$$\Theta(t) = \frac{2}{\sqrt{\pi}} \left(t - \frac{t^3}{3} + \frac{t^5}{10} - \frac{t^7}{42} + \dots \right),$$

and ($t < \sqrt{3}$)

$$\frac{6}{\sqrt{\pi}} \cdot \frac{t}{3+t^2} = \frac{2}{\sqrt{\pi}} \left(t - \frac{t^3}{3} + \frac{t^5}{9} - \frac{t^7}{27} + \dots \right),$$

$$\Theta(t) = \frac{6}{\sqrt{\pi}} \cdot \frac{t}{3+t^2} - \frac{2}{\sqrt{\pi}} \left(\frac{t^5}{90} - \frac{5t^7}{378} + \dots \right).$$

For small values of t the approximation is evidently good. Beyond $t = \sqrt{3}$ it clearly fails. But at $t = \sqrt{3}$ the error is within one per cent. At first the approximation is slightly in excess of the function, but the error reaches its maximum near $t = 1.2$, vanishes after $t = 1.5$, and then becomes negative. As $t = \sqrt{3}$ means more than 3.6 times the probable error, the range of useful approximation is quite considerable.

For some purposes it will be convenient to write

$$t = \sqrt{3} \tan \frac{1}{2}\phi, \quad \phi < 90^{\circ},$$

$$\Theta(t) = \sqrt{\frac{3}{\pi}} \sin \phi = [-0.0100] \sin \phi.$$

Thus for $\Theta(t) = \frac{1}{2}$, $\phi = 30^{\circ}.77$ and $t = 0.4766 \dots$ as compared with the true value $\rho = 0.4769 \dots$. Further comparison is given in the following table:

ϕ .	t .	t/ρ .	$\sqrt{\frac{3}{\pi}} \sin \phi$.	$\Theta(t)$.	Diff.
60°	1.000	2.097	0.8463	0.8427	+0.0036
69.8	1.208	2.533	0.9171	0.9125	+0.0046
83.1	1.535	3.219	0.9701	0.9701	0.0000
90	1.732	3.632	0.9772	0.9857	-0.0085

Beyond the last point the approximation allows for 23 cases per 1000, while the normal law allows 14. As abnormally large deviations are often in excess of expectation, this is no bad thing. For approximate calculations of this kind it is well to remember that the reciprocal of 2.1 is 0.4762, which is very nearly the value of ρ .

It is evident that

$$\int_0^t \Theta(t) dt = \frac{3}{\sqrt{\pi}} \log_e \left(1 + \frac{1}{3}t^2 \right) = 3.897 \log_{10} \left(1 + \frac{1}{3}t^2 \right)$$

is a very close approximation over the range $0 < t < \sqrt{3}$.

A far closer approximation to the error function has been given by Mr. H. B. C. Darling (*Quart. J. of Math.*, xlix. p. 36), but its form makes it unsuitable for the purely practical purposes here contemplated.

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Is *Orchis Fuchsii* (Druce) a Valid Species of Orchidaceæ?

THERE has for some time been going on a controversy among botanists as to whether Linnæus in establishing his species *Orchis maculata* omitted from, or intended to include within, it a very closely related form collected by himself, which has been raised to specific rank by Dr. Claridge Druce as *O. Fuchsii*. The question whether the latter is only a variant of