The Forces which lift Aeroplanes.[1]

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THE DIRECT GEOMETRIC ANALOGY.

THE fields of motion produced by the pulsating and the oscillating bodies are traced out by the registering instrument, and they can then be compared with the corresponding magnetic fields shown, for example, by iron filings. In this way it can be demonstrated that there is the most striking correspondence between the geometric structure of the hydrodynamic and the magnetic field (Fig. 2). The finest part of this geometric analogy, however, which concerns the simultaneously existing impressed and induced fields in the internal spaces, can only be recognised mathematically. There is the same striking resemblance between the hydrodynamic fields of permanent motion, and the corresponding magnetic fields of steady electric currents.

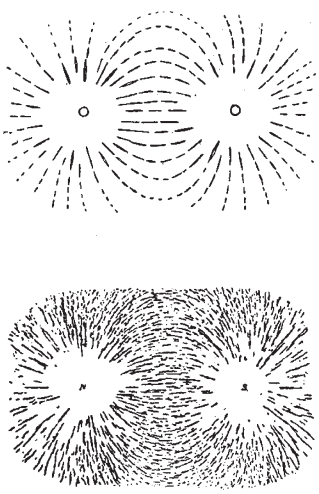

FIG. 2.—Example of the geometric analogy: bodies pulsating in opposite phases and magnetic poles of opposite signs.

One important reservation should be made, however, in connexion with this and the following experiments concerning permanent motion. The theory assumes a frictionless fluid while we make use of the fluid friction as a practical means for producing the required permanent circulation. We have brought in thereby an element not completely contained within our theory: this is important in order that we may recognise later the proper limitations of the conclusion derived from our theory.

THE INVERSE DYNAMIC ANALOGY.

We can also examine the mechanical forces in the two different kinds of field, and it is found that the hydrodynamic force produces attractions, repulsions, lateral displacements, and rotations equal but opposite to those which the corresponding magnetic force produces.

The analogy may be pursued in the most minute details. In all cases we have attraction by oscillations which are symmetrical, and repulsion by oscillations which are anti-symmetrical with respect to a plane, just as in magnetism we have attraction in the case of symmetry and repulsion in the case of anti-symmetry.

Passing on now from the analogy concerning vibratory motion to that of permanent motion, two cylinders rotating in the same direction repel each other so soon as they have had time to produce by friction the required circulating motion of the water. This corresponds to the attraction between electric currents in the same direction. Changing the direction of rotation of one cylinder, the repulsion changes into attraction as soon as the new motion has had time to develop itself. This corresponds to the repulsion between anti-parallel electric currents.

THEORETICAL AEROPLANE.

When my father had shown his experiments he was often asked if he could not imagine practical applications of them. He always answered, "That is not my affair." But if then the inquirer, half excusing his question, added, "The forces are perhaps too feeble to be of any use," he always got the answer, "From that point of view there is no objection. There is no limit to the strength of the forces."

Returning to this question now, forty to fifty years later, we at once see an application of the last illustration, namely, that with the rotating cylinders. The experiment succeeds equally well in air and water. The force will be reduced in proportion to the density, but for this we can compensate by increasing the speed of rotation. One cylinder is displaced normally to the wind produced by the other, or normally to a wind produced by any means, just as an electric current is displaced normally to a magnetic field however the field may be produced: the rotating cylinder is displaced towards that side where the circulation round it assists the external wind, while the current is displaced towards that side where its own field opposes the external field.

We can then use our rotating cylinder to ascend in the air. Given a horizontal wind, let the cylinder be placed horizontally and normally to the wind and rotated so that on its under side it moves against and on its upper side with the wind. Provided we prevent it from being carried along with the wind, it will then be pushed upwards as if attracted by an oppositely rotating cylinder higher up. Using the analogy we may calculate the force per unit length of the cylinder. To avoid unnecessary complications we use Heaviside's rational electromagnetic units. Then the hydro-electric current i equals the line-integral of the hydro-magnetic force, *i.e.* of the specific momentum, along a closed curve round the cylinder. Taking the constant density ρ of the air outside the integral sign we get $i=\rho\Gamma$, Γ representing the circulation round the cylinder. And multiplying by the hydro-magnetic induction, *i.e.* the velocity v of the wind, we get the force per unit length of the cylinder:

$$F=iv=\rho\Gamma v.$$

The formula will be exact if for Γ we take the circulation actually produced in the free air. But not knowing this circulation we may try as a first approximation to identify it with that of the circumference of the cylinder, fully recognising that we do not know to what extent friction is able to transfer this circulation to the air. Let then the cylinder have a periphery of 1 metre and a circumferential velocity of 1 metre per second. This will give unit circulation in the metre-ton-second system. If, further, the wind has the strength of one metre per second, and if we put the density of the air for simplicity equal to 1/1000, we have

[1] Continued from p. 474.

for each metre of the cylinder the lifting force of

$$\tfrac{1}{1000} \text{ M.T.S. force units} = \tfrac{1}{10} \text{ kilogram.}$$

A cylinder of ten metres length will then carry a kilogram. If we increase the circumferential velocity from 1 to 10 m. per second, the cylinder will carry 10 kg. If then we increase the wind velocity from 1 to 10 metres per second, it will carry 100 kg. Retaining the angular velocity and increasing the circumference of the cylinder from 1 to 10 metres, we get a lift of 1000 kg., and so on.

Instead of letting the cylinder mount in the wind we can move it against the resting air. We then get an aeroplane having a rotating cylinder instead of the aeroplane wings, and as our calculations have shown, it should be a very effective aeroplane from the point of view of lift if only a small fraction, such as the half or even the fifth part of the circulation of the cylinder, be transferred effectively to the air. But only experiments can give full information of this effectiveness.

The technical difficulties of such an aeroplane, however, are obvious. The main interest is, and will probably remain, theoretical: it is a mental instrument illustrating the dependency of flying upon the hydrodynamic actions-at-a-distance, just as Carnot's thermodynamic engine illustrates the dependency of the steam-engine upon the second law of thermodynamics. To see the relation of this, theoretically the simplest aeroplane, to the real aeroplane we must go to the results obtained by experimental work in the aerodynamic laboratories.

Automatic Production of the Lifting Circulation.

Here we meet with effects of friction which are not contained in the theory developed. We started with the paradoxical result that a spherical body moving at uniform velocity through a frictionless fluid experiences no resistance according to the solution of the hydrodynamical equations. What is then the origin of the resistance always experienced, which seems to be far too great to be explained merely as a direct effect of the small viscosity? The general reason has long been known: it is the formation of a wake of eddies. The origin of these vortices has been investigated especially by Prandtl and his school in the aerodynamical laboratory in Göttingen.

As demanded by theory, the vortices never originate in the free fluid. But however small the viscosity of the fluid may be, there will always be a thin layer close to the surface of the body in which the fluid masses are subject to an intensive shearing effect. This disturbs the symmetry of the motion which should otherwise exist between the front and the back of the cylinder (Fig. 3). Retarded fluid masses gradually accumulate behind the cylinder, forming there two vortices. From time to time these vortices are carried away, alternately from each side, producing the eddying wake (Kàrmàn's experiment). The resistance experienced by the cylinder is due to the work done in forming these eddies.

Now let us replace the circular cylinder by an elliptic one, set obliquely to the current. The symmetry being lost, the leeward and the windward vortices are formed under different conditions, and more especially, the conditions for their discharge are different. The windward vortex is carried away with the current, while the leeward remains anchored to the cylinder, forming a circulation round it. The lifting effect upon the inclined plane comes exclusively as a consequence of the circulation thus formed round it—a fundamental fact in aerodynamics first recognised by Lanchester, and put into a formula by Kutta and Joukowski. This formula is the same as that which we have just deduced from the hydrodynamic analogy to the electromagnetic field, and, as seen from this analogy, it is valid for all cross-sections of the wings and for circulations of any origin, not merely for the circular cross-section of the rotating cylinders in our theoretical aeroplane.

The difference between our theoretical aeroplane with its rotating cylinder and the practical aeroplane with its inclined wings reduces to this: with the rotating cylinder we produce systematically, in a way which

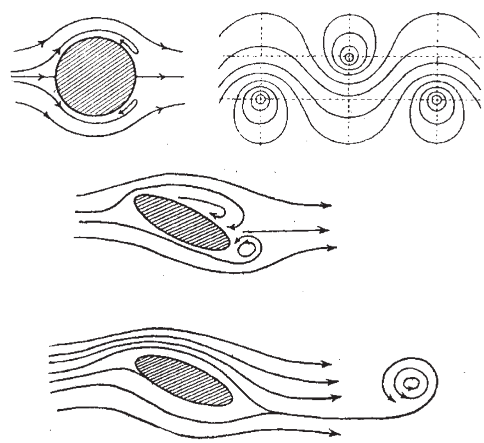

Fig. 3.—Different cylinders in a current.

we can control, the circulating motion which is the condition for lift. By the ordinary aeroplane wings the lifting circulation is produced spontaneously by the asymmetry of the wing.

Induced Resistance.

From the hydrodynamic analogy we have deduced the force lifting an aeroplane. In addition to the lifting force the aeroplane is subject also to head-resistance of which the analogy gives us the theory. Every vortex which the aeroplane leaves behind in the air represents a hydro-electric current which exerts its action-at-a-distance upon that main hydro-electric current which carries the aeroplane.

Fig. 4 shows diagrammatically the system of vortices which every aeroplane must necessarily leave behind itself: namely, the windward vortex which at the moment of starting is carried away from the aeroplane, and the vortices with axes parallel to the wind which join the ends of this vortex with those of the leeward vortex that remains anchored to the wing and carries the aeroplane. This gives a closed vortex having the form of a rectangle of which two sides are constant in length, while two increase with the velocity of the aeroplane relatively to the wind. What the aeronautical engineers call the induced resistance may be regarded as the hydro-electric attraction of that part of the vortex system which is left behind in the free air upon that part which is anchored to the wing. The analogy with electromagnetism allows us immediately to write down formulæ for this attraction.

The theory of hydrodynamic actions-at-a-distance, which has hitherto been so absolutely useless from a practical point of view, has recently become very practical. It opens up the possibility of applying electromagnetic formulæ to the theory of aeroplanes.

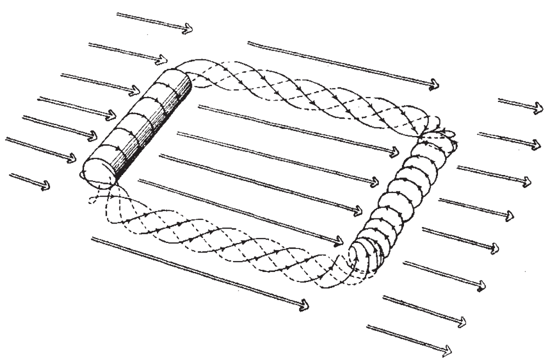

FIG. 4.—Vortex system set up by an aeroplane.

Moreover, it is a short step from the theory of the wing to the theory of the propeller. The fact that its motion is circular instead of translational is of no importance. The force exerted against the propeller blade depends upon the circulation produced round it, and can be calculated by the theory of the hydrodynamic actions-at-a-distance. From the propeller blade the way is not far to the turbine blade. The type of driving force will remain the same whether the driving medium be incompressible water or expanding steam. In the latter medium, effects of expansion will come into play of the kind exemplified by the experiment with the pulsating bodies. But the greater the complications, the more complete is the use which has to be made of the theory of the hydrodynamic actions-at-a-distance.

It is not least interesting from this point of view to consider the transfer of mechanical to electrical energy or vice versa. Let us suppose that at one end of a shaft we have a water or steam turbine, at the other end a dynamo. At the turbine blades the hydrodynamic actions-at-a-distance are in activity, and in the dynamo the equal but opposite electromagnetic actions. We derive both of them by the same formulæ, only with a change of sign when we pass from one set to the other. One set of forces is like a reflected image of the other.

Do we not here behold a deep harmony of Nature at a point where important practical problems are intertwined with ideas of deep theoretical interest? Finally, what does Nature mean by placing us face to face with this wonderful harmony between such different branches of physics as hydrodynamics and electrodynamics? This is a question which may be answered by a future Faraday.

Obituary.

VISCOUNT LONG OF WRAXALL, F.R.S.

LORD LONG of Wraxall, whose death occurred on September 26 at seventy years of age, was well known to British workers in several scientific fields. He will be particularly remembered for the unswerving support which he gave to scientific advisors while president of the Board of Agriculture in 1892–1900, as regards the necessary measures to extirpate rabies from the British Isles. At that time it had been proved beyond doubt that hydrophobia was a specific infective disease which did not originate *de novo*, but could only be introduced into a district or country by being passed on from animal to animal. Acting upon this and other knowledge when president of the Board of Agriculture, Lord Long was responsible for the issuing of muzzling orders for dogs, first for London and then for the whole country. The National Canine Defence League thereupon instituted a public campaign against the muzzle and condemned the measures wisely adopted by the Board. Lord Long, however, had instructed himself thoroughly well in the whole question of rabies, and, with laudable firmness, he resisted the outcry and the repeated assaults of the uninstructed sentimentalists. As a result, he was able to demonstrate that, by the strict carrying-out of muzzling orders, rabies could not only be checked but also reduced eventually to extinction. The eighty thousand "dog-lovers" who petitioned for Lord Long's dismissal from his office at the Board of Agriculture showed themselves to be poor friends of dogs by their action, which many of them must afterwards have regretted. In recognition of his work on behalf of science, Lord Long was elected a fellow of the Royal Society in 1902. He was also an honorary LL.D. of the University of Birmingham.

DR. R. S. WOODWARD.

DR. ROBERT SIMPSON WOODWARD, formerly president of the Carnegie Institution of Washington, died on June 29, aged seventy-four, and the following account of his scientific work is taken from the Journal of the Washington Academy of Sciences. Dr. Woodward was born at Rochester, Michigan, July 21, 1849. Following his education as a civil engineer at the University of Michigan, he served with the U.S. Lake Survey, the Transit of Venus Commission of 1882, the U.S. Geological Survey, and finally the U.S. Coast and Geodetic Survey. Leaving the Federal service in 1893, he became professor of mechanics and mathematical physics at Columbia University, New York City. In 1905 he succeeded the late Daniel Coit Gilman as president of the Carnegie Institution of Washington, then but recently founded by Andrew Carnegie. Dr. Gilman's term of office as its first president had been very short, and the real responsibility for formulating the working plans for the development of a new and comparatively untried form of research institution fell upon Dr. Woodward. Following fifteen years of successful administration in this office he retired from active duty in January 1921. He was president in 1900 of the American Association for the Advancement of Science and had also served as president of the American Mathematical Society, the Washington Academy of Sciences, and the Philosophical Society of Washington. He was a member of the National Academy and other national organisations. Dr. Woodward made notable contributions to mathematical physics and astronomy, especially as applied to geodesy and geophysics.