

to do is to make the ratio $r : R$ exactly equal to $\tan \pi/n$, where n is any integer, and we shall get the harmonic sequence

$$1, n-1, n+1, 2n-1, 2n+1, 3n-1, 3n+1, \text{ etc.}$$

For pedal notes the best values of n are 5, 7, or 9; these will give octave harmonics. If we take $n=5$ we shall get a pipe shorter than the stopped diapason of the same pitch in the ratio of 4 : 5; but its harmonic overtones will be the series

$$1, 4, 6, 9, 11, 14, 16, \text{ etc.},$$

which should give a far better quality than the ordinary stopped diapason.

But the best of all for quality is the $\pi/3$ *Bicylindron*, the length of which is two-thirds that of the open diapason of the same pitch. This pipe has the harmonic sequence

$$1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, \text{ etc.}$$

This pipe, if properly voiced, should make a solo stop with a unique loveliness of tone. Experimentally I have proved the sequence 1, 2, 4, and 5.

I worked out my formulæ in the case of a double conical pipe, made of two cones joined together at the broader equal ends, each tapering to a smaller mouth in opposite directions. The result can be found by a very pretty calculation from the formulæ given by Basset or Lord Rayleigh; it is

$$m \sin mH = \left(\frac{1}{h} + \frac{1}{h'} \right) \sin mh \sin mh',$$

where $H = h + h'$; h and h' have the meaning given above for the *Pyramidon*; h' and h' the corresponding quantities for the second cone; and m gives as before the vibration-rate of the whole pipe, which is of lower pitch than an open cone of the same length. This double pipe is intended for reed-stops, which need a cone tapering to the narrow reed-tube. By taking $h' = h$ or $h/2$ we can get all the harmonics of the open cone h' ; and by giving suitable values to h and h' we can ensure that the fundamental tone shall be an exact double octave below the open cone h' , thus gaining the conditions for good quality. Hence we find in either case

$$\frac{1}{h} + \frac{1}{h'} = \frac{\pi}{2h}$$

In 1868 I made four pipes on this plan, using some harmonium reeds I had by me, of pitch Tenor C, D, E, and F. Mr. Frye tried them in the Cathedral organ. We both agreed that the tone was very beautiful, when it came on. But there was a distinct burr at the beginning and end of the tone, and the pipes were so "slow of speech" that in a quick run they never spoke at all. Such "free reeds" are probably inappropriate: they are never used by English organ-builders. But the experiment shows that in expert hands a reed-pipe of this kind might be made with a tone of unique beauty, free from all defects of "speech." JAMES A. ALDIS.

The Quantum Theory of Dispersion.

In a recent letter to NATURE (May 10, p. 673) Dr. Kramers advanced a quantum theory of dispersion which is a generalisation of the theory of Ladenburg. The formula proposed by Kramers for the polarisation of an atom when put in a wave is his formula (5). This formula is stated by Kramers to satisfy the condition demanded by the Correspondence Principle, namely, that the dispersion due to an atom in a state of high quantum number is the same on the classical

and on the quantum theories.¹ The presence of the second term has been introduced by Kramers for this purpose. From the point of view of the virtual oscillators of Bohr, Kramers, and Slater, the second negative term of Kramers is somewhat dissatisfying, because an oscillator would give rise only to a term of the first positive type.

The present writer has been also considering the question of interaction between radiation and quantised atoms in connexion with the question of the Brownian movement of atoms in black body radiation. A picture similar to the virtual oscillator finds application also in that field. However, the exact form of the interaction has been conjectured by the writer to be somewhat different from that proposed by Kramers.

The difference can be illustrated in the case of the linear oscillator. In this case the expression of Kramers becomes at long wave-lengths

$$P = [n - (n-1)] \frac{e^2}{m} \frac{1}{4\pi^2(\nu_i^2 - \nu^2)} = \frac{e^2}{m} \frac{1}{4\pi^2(\nu_i^2 - \nu^2)}.$$

The same result may be also derived as

$$P = \left[\frac{1}{2} + \frac{1}{2} \right] \frac{e^2}{m} \frac{1}{4\pi^2(\nu_i^2 - \nu^2)}.$$

In this manner the negative term may be avoided. In order to satisfy the Principle of Correspondence, the dependence of P on ν must be in general slightly more complicated than that for the oscillator. This dependence can be derived from a consideration of a "virtual orbit" rather than a virtual oscillator. (I am indebted to Prof. Van Vleck for this term.) The "virtual orbit" has the same frequency as the "virtual oscillator." However, its reaction to the external field is comparable with that of an electron, the orbit of which is the mean of all the orbits between the two stationary states. (The meaning of "mean" is of necessity somewhat indefinite.) It is clear that in the general case a properly taken sum of the contributions of the various ν_i^a, ν_i^e will give the required result. Thus it is sufficient to attribute to a transition between a quantum state of quantum number $(n_1 + \tau_1, \dots, n_u + \tau_u)$ to (n_1, \dots, n_u) one half of the contribution to the polarisation on the classical theory due to the terms in frequencies $\tau_1 \omega_1 + \dots + \tau_u \omega_u$ (in Bohr's notation) in order to satisfy the Principle of Correspondence.

It appears that the form of the theory here outlined is better capable of explaining dispersion at long wave-lengths (say for an atom in the normal state) than the form of Kramers because the characteristics of the motion are of greater influence on the "virtual orbit" than on the "virtual oscillator" point of view. By introducing a properly taken mean, one may hope to obtain the influence of a static field as a limiting case of a field considered in the theory of dispersion. G. BREIT.

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THROUGH the courtesy of the Editor of NATURE, I have been permitted to see Mr. Breit's letter, and I welcome the opportunity thus afforded me to add some further remarks on the theory of dispersion, in order to elucidate some points which were only briefly touched upon in my former letter.

In addition to the empirical applicability of a dispersion formula of the type (4), the arguments which

¹ Prof. J. H. Van Vleck, of this University, has shown, in a publication which is to appear soon, that this formula satisfies the Correspondence Principle for the case of the general non-degenerate multiple periodic orbit.