There are two positive roots, both between 0 and 1. The greater is the more suitable for our series. It is very nearly equal to unity and it was found to be very nearly $32/33$.

As a second approximation $n!$ is then equal to

$$\sqrt{2\pi} \left(\frac{n+(1/a)}{e} \right)^{n+(1/a)} \left(n + \frac{1}{a} \right)^{(a-2)/2a} \left[1 + \frac{1/6 + (1-a)/a^2}{2\{n+(1/a)\}} \right], \quad (5)$$

where $1/a = 32/33$.

From this expression, which is affected by an error of order $-1/360n^3$, $10!$ was calculated:

$$10! = 362,8806 \text{ (an error of 6).}$$

The approximation (4) gives

$$10! = 362,8787 \text{ (an error of 13).}$$

From the original value of a , the second approximation will be

$$n! = \sqrt{2\pi} \left(\frac{n+b}{e} \right)^{n+b} (n+b)^{-c} \left\{ 1 - \frac{.0080,1875}{(n+b)^2} \right\}. \quad (6)$$

The error in this case will be less than $1/2000 \times n^3$ of the whole.

(6) gives

$$10! = 362,8801 \text{ (an error of 1).}$$

Forsyth's approximation (4) has an error of order $1/240n^3$. It will be seen that the first approximation (3) is a remarkably good one, and the expression is quite good for calculation purposes. The value of $n!$ may be calculated in a very short time.

Mr. Soper's expansion, taken to the same order as (4), gives

$$10! = 362,8792 \text{ (an error of 8),}$$

with an error of order $1/400n^3$. The second approximation (6) derived in the same way as our first approximation is exceedingly accurate, and is better than that of (4); it is also better than Mr. Soper's, which in turn is better than Prof. Forsyth's (4).

Prof. K. Pearson has given in *Biometrika*, vol. vi., a very close approximation to the value of $n!$. This takes account of terms up to $1/n^4$ and partially of the term in $1/n^5$:

$$\log \frac{\Gamma(n+1)}{n^n e^{-n}} = 0.3990899 + \frac{1}{2} \log n + .080,929 \sin \frac{25^\circ.623}{n}. \quad (7)$$

On evaluating $10!$ by means of this expression, it is found that the exact value is given to the nearest unit.

My chief aim in this note has been to show that a very good first approximation may be obtained without the use of any terms of the exponential and that the resulting expression is useful for computing factorials.

It may be of interest to give the values of $1! 2!$ and $10!$ found from these approximations in a single table:

	(3) of Present Note.	(4) Forsyth.	(7) Pearson.	Exact.
1!	1.00248	.99883	.99952	1.00000
2!	2.00266	1.99948	1.99996	2.00000
10!	362,9051	362,8784	362,8800	362,8800

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Dr. Kammerer's Alytes.

MAY I reply in a few words to Dr. Bateson's brief letter on Kammerer's Alytes, which appeared in NATURE of June 30?

Dr. Bateson states that when the nuptial callosities of genera allied to Alytes are described as appearing on the "inner" sides of the fingers, the word "inner" means the radial side and not the palmar surface.

This is quite true, but the callosity on the radial edge of the finger involves the palmar surface also, as Dr. Bateson may convince himself by inspecting Boulenger's figures, and as, indeed, is demonstrated to every student when he is shown the nuptial callosity of the male Rana.

Further, I learn from a letter from Dr. Kammerer that in the specimen of Alytes shown at the Linnean Society, the callosities extend round the radial edges of the fingers on to the dorsal surface, and that he would have demonstrated this to any one who had raised this point while he was explaining his specimens before the meeting.

Readers of NATURE are thus now in a position to judge what ground there was for Dr. Bateson's objections.

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Molecular Interruption.

IN reply to Mr. R. d'E. Atkinson's criticism (NATURE, March 10, p. 326) of my note on the possibility of selective molecular interruption, I should like to point out that so far from attempting to dispose of the validity of the ordinary treatment and claim the effect in question for "infinite free path," I had already shown (NATURE, July 22, vol. 110, p. 112) the reverse to be the case, and that such an effect is not then possible.

It is manifestly clear that it is illogical to conclude, however, because this is the case with "infinite free path" (i.e. in the absence of intermolecular collision in the system), that it must also be true for a system in which intermolecular collisions exist, with long free paths relative to the diameter of the directing vessel employed, the particular and special case alone dealt with in my note.

Mr. Atkinson's misinterpretation appears to have arisen from his overlooking my words "molecules issuing from collision in circle O," since his statement "all points on their long paths may equally be taken," as being in O is otherwise unintelligible.

His statement that I have admitted the length of the free path to be irrelevant is not correct. The excessive downward bias to which he refers is, in my opinion, due entirely to the fact that molecules proceeding from collisions (with equal probability of motion in all directions) are interrupted by the vessel before the end of their normal free path period, when they are moving in certain specific directions; and are uninterrupted throughout the whole of their normal flight, when they are moving in other specific directions: a selective redirection or elimination of the former class which must continuously be leaving a corresponding preponderance of the latter—a conclusion which more careful calculation confirms.

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