Letters to the Editor.

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The Spectrum of Neutral Helium.

IN a letter to NATURE of November 25, p. 700, concerning my first communication (NATURE, August 19) on this subject, Prof. Raman expresses the opinion that the representation of helium lines derived from my assumption of the mutual apathy of the two interatomic electrons has a purely arithmetical character and would thus be deprived of any "real physical basis."

Disregarding a number of remarks reducible to the laceration of the empirically established series, a regrettable feature pointed out by myself (*Astrophys. Journ.*, September 1922), it will be enough to reply here to Prof. Raman's chief and apparently strongest objections. These are two: first, that the numerous coincidences yielded by my formula are simply explicable as so many "fortuitous arithmetical coincidences," and, second, that the particular value (109723) of the Rydberg constant N used in that formula is, in general, inadmissible, this value belonging legitimately to He^+ , with a single electron, but not to the neutral atom with its two electrons.

Now it so happens that precisely these two points have steadily occupied my attention since the formula was first published, and I am therefore able to reply to both of them without delay. The corresponding details of reasoning and numerical data being all given in a paper just communicated to the American Physical Society (for its Boston meeting, December 27-29), to which readers may be referred, it will suffice to describe here the bare results.

1. Consider only those lines for which the final quantum numbers lie between 3 and 8, and the initial ones between 4 and 20, which fall within the interval $\nu = 17,000$ to 37,850. The total number of such distinct lines is $n = 680$. Among these there are $k = 45$ lines covering observed helium lines, with a mean deviation $|\delta\nu| = 2.57$. In the considered ν -interval there are in all 97 observed lines. Whence, the mean (geometric) probability of hitting an observed line in a single trial by mere chance, $p = 0.0182$, and the probability P of hitting k or more such lines in n (680) trials, by Bernoulli's theorem, $P = 1 - \Theta(x)$, where $\Theta(x)$ is the error-function and

$$x = \frac{\Delta}{\sqrt{2pn(1-p)n}}, \quad \Delta = |k - pn|.$$

In our case $pn = 12.40$, $\Delta = 32.60$, and therefore the probability that our set of 45 coincidences should be "fortuitous" is

$$P = 1 - \Theta(6.61),$$

which is a little less than $1.7 \cdot 10^{-13}$ —small enough to discard every suggestion of the play of blind chance. This conclusion is considerably strengthened when other groups of coincidences tabulated in the *Astrophys. Journal* are similarly treated.

Of particular interest, in this and other respects, are the 18 lines of the type $\left(\frac{m_1 \cdot m_2}{4.8}\right)$, and another group of three lines, each of the type $\left(\frac{5 \cdot m}{4.4}\right)$ and each covering an empirical "combination" line of the "doublet system."

2. Let m be the mass of each of the two electrons,

M that of the nucleus, and $\epsilon = m/M$. Taking account of the wobbling of the nucleus, through which the otherwise indifferent electrons perturb each other indirectly, and rejecting terms in ϵ^2 , etc., the energy of the system in any stationary state, say $n_1 = \iota$, $n_2 = \kappa$, is found with comparative ease. This divided by ch gives the corresponding "term," say $T_{\iota\kappa}$, our ν being the difference of two such terms. If this be written

$$T_{\iota\kappa} = 4N_{\iota\kappa} \left(\frac{1}{\iota^2} + \frac{1}{\kappa^2} \right),$$

then $N_{\iota\kappa}$, the "Rydberg constant" belonging to the particular pair $(\iota\kappa)$ of electronic orbits, is a certain symmetrical function of the integers ι , κ , and of the mutual orientation of the two orbits. For the case of quasi-circular orbits (*i.e.* such as would become circular for $\epsilon = 0$ or no wobbling) the investigation given in the Boston paper leads to the interesting result

$$N_{\iota\kappa} = N_\infty \left[1 - \epsilon - \epsilon\gamma \frac{\iota^6 + \kappa^6}{(\iota^2 + \kappa^2)^2 \kappa^2} \right], \quad (1)$$

where N_∞ (about 109737) is the constant for $M/m = \infty$, and γ the time-average of the cosine of the angle between the radii vectors of the two trabants. This formula holds for any inclination (i) of the two orbits and for any phase difference (a) of the two electrons describing them.

Now, a purely kinematical reasoning gives for $\iota \neq \kappa$ the value $\gamma = 0$, and for $\iota = \kappa$,

$$\gamma = \gamma_{\kappa\kappa} = \frac{1}{2} \cos a (1 + \cos i), \quad (2)$$

where a is the angular distance of one electron from the ascending node when the other electron just passes through it.

Since for $\iota = \kappa$ the arithmetical expression in (1) becomes equal to unity, we have

$$N_{\kappa\kappa} = N_\infty [1 - \epsilon - \epsilon\gamma_{\kappa\kappa}], \quad (1a)$$

which, by (2), can assume any value from N_∞ down to $N_\infty(1 - 2\epsilon)$, with $N_\infty(1 - \epsilon)$, the desirable He^+ value, just in the middle of the interval. If, *e.g.*, the orbits are coplanar and $a = 180^\circ$, we have $N_{\kappa\kappa} = N_\infty$, for then there is no wobbling; if $a = 0$, $N_{\kappa\kappa}$ would reach the other extreme value, about 109709, and for $a = 90^\circ$ we should have the mean value (109723), which might even be made the only value if the lines of the type $\iota = \kappa$ are not to be very broad. There is thus no essential difficulty. Moreover, very few among my tabulated lines have $\iota = \kappa$.

For the overwhelming majority of those lines we have $\iota \neq \kappa$, when γ vanishes, and (1) becomes, no matter what the inclination of the orbits and the phase difference of the electrons,

$$N_{\iota\kappa} = N_\infty (1 - \epsilon), \quad (1b)$$

which is precisely the value (109722 to 23) used in my formula. This in itself seems to be a strong support for that formula.

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December 6.

Returning to my letter of December 6, I beg to supplement the same by a result of my last week's work, which seems to give the proposed theory a much stronger support than all probability estimates, for it represents *in toto* and orderly some empirical series of helium. In fact, guided by a few coherent items of my original table, I find that the whole diffuse series of singlets, denoted by $1P - mD$, is represented by

$$\nu = 4N \left(\frac{18.2n}{20.4} \right) \equiv N \left(\frac{9 \cdot n}{10.2} \right),$$

two final and one initial quantum numbers being