observers X and Y. The spaces which they use are normal to these axes. Then if v be their mutual relative velocity,

$$v = \tanh C,$$

the velocity of light being unity.

It may be added that the relation $B + C = A'$ is a particular case of the more general "triangle of relative velocities." Let OP, OQ, OR be a triad of co-directional non-coplanar temporal vectors (Dr. Robb's "inertia lines") cutting the "open hyper-sphere" (centre O)

$$u^2 - x^2 - y^2 - z^2 = 1$$

in point-instants P, Q, R, where u is the time co-ordinate. Let a, b, c be the geodesic arcs QR, RP, PQ *within* the hyper-sphere. These arcs are minima, not maxima; their elements in the limit are spatial in character, being normal to time-vectors; their hyperbolic tangents represent the mutual relative velocities of observers (X, Y, Z) who use OP, OQ, OR, or parallels thereto, as their time-axes. The Euclidean space used by X at any instant is parallel to the tangent space at P to the hyper-sphere, and from the point of view of X the directions of the relative velocities of Y and Z are the tangent-lines at P to the geodesic arcs PQ, PR. The angle between these directions is a circular angle (P), and the metrics of the geodesic triangle PQR are contained in the formulæ

$$\cosh a = \cosh b \cosh c - \sinh b \sinh c \cos P,$$

$$\frac{\sin P}{\sinh a} = \frac{\sin Q}{\sinh b} = \frac{\sin R}{\sinh c}.$$

When a, b, c are very small compared with the radius of the hyper-sphere the spaces of the observers are regarded as parallel, and we get the ordinary formulæ

$$a^2 = b^2 + c^2 - 2bc \cos P, \text{ etc.}$$

When OP, OQ, OR are coplanar we get the relation as before (with change of letters)

$$a = b + c.$$

The above remarkable formula for relative velocities was, I believe, first discovered by Dr. Robb, and is set forth by Dr. Weyl ("Space, Time, and Matter," § 22). I am not aware, however, that its direct connexion with the geodesic geometry of the open hyper-sphere has been explicitly noticed. R. A. P. ROGERS.

Trinity College, Dublin, October 31.

Space-Time Geodesics.

IN NATURE of October 28, p. 572, Dr. Robb pointed out the incorrectness of asserting that the length of a "world-line" is a minimum between any two points of it. He gave an example in which the length was neither a minimum nor a maximum. The object of his letter, no doubt, was to remind some reckless relativists that they should be more careful in their language. But there is the danger that some may suppose that he was dealing with a real weakness in Einstein's theory. To dispel this idea we may recall a few well-known facts.

Treatises on the geometry of surfaces (in ordinary three-dimensional Euclidean space) define *geodesics* in various ways. Some say that a geodesic is the shortest line that can be drawn on the surface between its two extremities, and they use the calculus of variations to find its equations. This method is open to criticism. The researches of Weierstrass have shaken our faith in the infallibility of the results obtained by an uncritical use of the routine processes of that calculus. But whatever may be said against the process employed, the equations of a geodesic finally obtained agree with those obtained by more trustworthy methods. For example, we may define a geodesic as a curve such that at every point the osculating plane is perpendicular to the tangent plane to the surface. From this definition we can easily obtain (cf. Eisenhart's "Differential Geometry," p. 204) equations which in the usual abbreviated notation of tensor calculus may be written

$$\frac{\partial^2 x_\sigma}{\partial s^2} + \left\{ \alpha\beta, \sigma \right\} \frac{\partial x}{\partial s} \frac{\partial x_\beta}{\partial s} = 0, \quad (\sigma = 1, 2).$$

Einstein's equations ("The Meaning of Relativity," p. 86) are the obvious generalisation of these and differ merely in that the suffixes range over the values 1, 2, 3, 4, instead of only 1, 2. His notation is slightly different from the form given above, which is due to Eddington.

These equations can be obtained by at least two other methods. Einstein uses a "parallel displacement" method due to Levi-Civita and Weyl. Eddington ("Report on the Relativity Theory of Gravitation," p. 48) shows that the equations are satisfied (or not) independently of the choice of co-ordinates, and that they reduce to the equations of a straight line for Galilean co-ordinates. This straight line is described with uniform velocity, so Einstein's equations may be regarded as a generalisation of Newton's first law of motion.

Applying these equations to the example given by Dr. Robb, we find that his space-time curve does not satisfy them unless $F''(x) = 0$. This means that $F(x)$ must be a linear function of x and so it cannot fulfil the required conditions of vanishing for two different values of x, except in the trivial case $F(x) = 0$. Thus the ambiguity seems to lie, not in Einstein's equations of motion, but merely in a particular method of arriving at them.

As regards the desirability of modifying Einstein's ideas on the nature of time, it is hazardous to give a definite opinion at present. It may be noted that Prof. Whitehead's new book ("The Principle of Relativity") endeavours to combine all the verifiable results of Einstein's theory with somewhat conservative ideas concerning space and time. The modified theory leads to some remarkable predictions (p. 129) which should be tested by experiment.

H. T. H. PIAGGIO.

University College, Nottingham,
November 4.

The Dictionary of Applied Physics.

THE issue of NATURE of September 30, p. 439, contained a highly appreciative review of the first volume of the "Dictionary of Applied Physics," and, as editor, I am much indebted to the author for his kind words. One remark, however, has, I gather, led to some misunderstanding; may I have space for a brief explanation?

Dr. Kaye directs attention to some of the "omissions," with the view of their future rectification. Most of these "omissions" will be found dealt with in future volumes of the Dictionary. Thus, in an article in vol. iii., on Navigation and Navigational Instruments, by Commander T. Y. Baker, the gyro-compass is treated of very fully, while, in vol. v., Mr. Dobson has a highly interesting article on instruments used in aircraft.

It has been part of my plan to separate the mathematical treatment of a subject and its practical applications. In this manner I hoped to increase the utility of the work to various classes of readers, some of whom are interested chiefly in the theory, while others are more closely concerned with the more practical details. R. T. GLAZEBROOK.

5 Stanley Crescent, Kensington Park Gardens,
London, W.11.