word "geometry" is commonly understood, *e.g.* by the Board of Education.

One remark as to Newton and the apple, which I intended to typify a supported observer and a continuously falling observer, respectively. If, with Mr. Cunningham, we take the apple to typify an observer at first supported and afterwards free, the apple's view of things is appallingly complicated—compared even with Newton's. But that only the more emphasises the point that the natural simplicity of things may be distorted *ad libitum* by the process of fitting into an unsuitable space-time frame.

A. S. EDDINGTON.

Observatory, Cambridge,
November 3.

I AM obliged to the Editor for giving me an opportunity to add a few words in comment upon Prof. Eddington's letter, and I do so in no captious spirit, but because it seems to me that in these very fundamental discussions it is of the utmost importance to clear away as many misunderstandings and difficulties as possible; to recognise that some divergences are merely consequences of viewing the same matter from different points of view, but that others may be due to looseness of thought on one side or the other; and I am glad to be able to recognise that most of the divergence of Prof. Eddington's exposition of the meaning of Einstein's theory from my own understanding of it is merely part of the difference between our natural ways of thinking. But two sentences in Prof. Eddington's letter do sum up my difficulty in regard to his exposition so clearly that I would like to direct attention to them.

"He admits, however, that all measurements that have ever been made are contained in the picture, and, I might add, all measurements that ever will be made. Thus we have a large number of measured intervals available for discussion."

In this sentence Prof. Eddington begs the whole question with which I ventured to end my review of his lecture. All measurements of length and all measurements of time that were ever made are, I agree, in the picture. But who ever measured this physical "interval"? What is the absolute scale of interval, and how is it applied? Again in Prof. Eddington's letter we read: "Clearly if a wrong geometrical system is used, the *measured* intervals will expose it by their disagreement." Unfortunately this is not at all clear to me, and I will try to explain why. So far as I can see, all actual physical measurements are records of observations of coincidences, *e.g.* of marks on a scale with marks on another body. That is to say, they correspond to intersections and concurrences of world lines of distinct physical elements. The significant feature of the four-dimensional picture of the universe is therefore merely the order of arrangement of such concurrences along the world lines of these physical entities. All else is of the nature of an arbitrarily adopted method of description of these orders of arrangement and is not contained in the picture itself. A geometrical system is an analytical means of describing the picture. The concurrences remain and their order is unaltered, no matter how we change our geometrical system. If I adopt a geometrical system other than that of Einstein, I may find the mathematics more complicated, but the actual observable facts recorded are the same—just as the fact of the meeting of the Great Northern, Great Eastern, Midland, and London and North-Western Railways in Cambridge station is quite independent of any particular brand of map or time-table. Of course a map which denied this fact would be wrong—but the adoption of a different geometrical system of attaching what I must not call "interval" to the separateness of two events does not break up a concurrence. It is just because *actual measurements* will not be altered by any change of the geometrical system that I cannot agree with the sentence I have quoted.

E. CUNNINGHAM.

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November 11.

The Time-Triangle and Time-Triad in Special Relativity.

DR. ROBB directs attention in NATURE of October 28, p. 572, to the fact that there is much confusion of thought with regard to the stationary value of the integral $\int d\sigma$ in the special theory of relativity. When the path is purely temporal, as Dr. Robb was the first to point out, the integral is an absolute maximum, not a minimum. Prof. Eddington has also directed attention to this truth. The following view may be of interest. I give mainly the results, as the precise mathematical proof would occupy too much both of space and time.

Let A, B, C be the vertices (point-instants) of a *pure time-triangle* in the field of special relativity. Suppose C precedes A, and A precedes B *in proper time*; then it may be proved that C precedes B, *i.e.* proper time order is *transitive*. Then if *cosh* C denotes the unit-scalar product of the vectors CA, CB, and if α, β, γ denote the real and positive intervals BC, CA, AB, we have

$$\cosh C = \frac{\alpha^2 + \beta^2 - \gamma^2}{2\alpha\beta}.$$

It may be proved that the expression on the right-hand side is always positive and is greater than unity. Thus C may be regarded as the real invariant "*hyperbolic angle*" between the temporal vectors CA and CB. This angle has the same metrical value for all observers moving with uniform mutual relative velocities.

It can also be proved that $\alpha > \beta$. Hence, since $\cosh C > 1$,

$$\alpha > \beta + \gamma.$$

That is, *the greatest side of a pure time-triangle is greater than the sum of the other two sides.*

It follows at once that the stationary value of the integral $\int d\sigma$, where the path is purely temporal, is an absolute maximum.

There is thus a real hyperbolic angle between any two co-directional temporal vectors. The triangle ABC has two real "internal" hyperbolic angles (B and C), and one real "external" hyperbolic angle A′. Besides the above formula we have

$$\cosh A' = \frac{\alpha^2 - \beta^2 - \gamma^2}{2\beta\gamma}, \quad \cosh B = \frac{\gamma^2 + \alpha^2 - \beta^2}{2\gamma\alpha}.$$

Taking positive signs for intervals and angles, we have

$$\frac{\sinh A'}{\alpha} = \frac{\sinh B}{\beta} = \frac{\sinh C}{\gamma}$$

and $\cosh (B + C) = \cosh A'$.

Thus *the one real external angle of a time-triangle is equal to the sum of the two real internal angles.*

The hyperbolic angle between two co-directional temporal vectors has a perfectly definite physical meaning, if the physics of special relativity is sound. Let CA and CB be the time-axes used by two