Letters to the Editor.

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Generalised Lines of Force.

LET me direct attention to a notable paper by Prof. E. T. Whittaker, in the Proceedings of the Royal Society of Edinburgh for November last.

It has often been discussed whether electric or magnetic lines of force were the more fundamental, and which might be regarded as the "cause" of the other; a discussion rather like the old controversy as to the direction of vibration in a polarised beam of light. Maxwell's theory satisfied the disputants by making both directions equally important.

In this paper Whittaker extends Faraday's theory of lines of force to electromagnetic activities in general, expressing the complex facts by aid of the space-time continuum of Minkowski, whereby any form of kinematics can be expressed as a sort of four-dimensional statics. The solenoids of Faraday are shown to be special cases of a more general kind of surfaces, called *calamoids*, which reduce to ordinary electric lines of force when the field is purely electrostatic, while they reduce to ordinary magnetic lines of force when the field is purely magnetic; so that the Faraday electric and magnetic lines are not two distinct and (as it were) rival things, but two limiting cases of the same thing. The essential solenoidal condition—strength inversely as cross-section—is retained if, instead of electric or magnetic force separately, the contra-variant $\sqrt{(E^2 - H^2)}$ is employed, and if the cross-section is that of a calamoid.

Ordinary equipotential surfaces, whether electric or magnetic, are seen to be special cases of an "electro-potential" or "magneto-potential" [? ether-potential] surface, which reduces to one or other of them whenever the field becomes static. These electro-potential surfaces, in general, exist in the four-dimensional world of space-time; but when the field is static each surface is wholly contained within three-dimensional space, and is an ordinary equipotential surface.

The property of the Faraday lines of force, that they are everywhere perpendicular to the equipotential surfaces, is shown to be a case of the more general theorem that the calamoids are everywhere "half-orthogonal" to the electropotential surfaces; (half-orthogonality being the four-dimensional analogue of three-dimensional perpendicularity).

In electrostatics, the total strength of all the Faraday tubes which issue from a closed surface containing no electric charge is zero; similarly, in general radiation-fields, the total strength of all the calamoids which cross a closed surface is zero. This theorem provides an intuitive geometrical integration of the Maxwell-Lorentz equations of the electromagnetic field.

There are also elaborated generalised "divergence" and "curl" theorems, with a certain kind of absoluteness about them, since they are independent of the motion of any observer.

For much fuller and more trustworthy information Prof. Whittaker's paper must be referred to. He said something about it in Section A at the recent meeting of the British Association in Edinburgh, but I, for one, did not understand his meaning then, in the rush of Sectional procedure. The object of the present summary is merely to direct early attention to a paper which cannot be long overlooked.

December 28.

OLIVER LODGE.

NO. 2725, VOL. 109]

Units in Aeronautics.

WRITE the usual formula in aeronautics,

$$\frac{R}{S} = 23.7 \left(\frac{V}{100} \right)^2, \text{ lb/ft}^2,$$

in the Hospitalier notation, and the practical airman recognises that the resistance R , lb, over a surface S , ft², is at the rate 23.7 lb/ft² at a normal velocity of 100 f/s; and so on for any other velocity V , on the law of the square.

The airman pays no heed to any units except his foot and pound, and he has no use for any of the elaborate explanations of Mr. A. R. Low.

The factor 23.7 will be the result of experiment in the air-channel, reduced for air of standard conditions of barometer and thermometer on the ground.

But the air density is never measured in any of these experiments, and it is doubtful if the measurement has ever been carried out in any aeronautical laboratory.

In the early history of the Royal Society "weighing the air" was a favourite research. Charles II. bet Buckingham fifty guineas to one he would demonstrate the compression of air in his hollow walking-cane; but the other story of the fish in a bucket of water cannot be traced further back than Whateley, who is supposed to have invented it.

The air density arises in the formula of the treatise on aerodynamics on the idea that the formula is the expression of Newton's assumption that the resistance is due to the impact of inelastic air particles, as if air could be treated as a cloud of dust; and then at an air density w , lb/ft³, or better for calculation in thermo-aerodynamics, at a specific volume the reciprocal $C = 1/w$, ft³/lb, Newton's formula becomes

$$\frac{R}{S} = w \frac{V^2}{g} = \frac{V^2}{gC}, \text{ lb ft}^2; \text{ or } \frac{R}{S} = 2wh, h = \frac{V^2}{2g}.$$

In this treatment the Equation of Continuity is ignored; the air particles should stop dead and fall down in a heap at the foot of the aeroplane, to be swept up as dust.

Thus the mysterious factor 0.00237 of the treatise on aerodynamics is the equivalent of w/g , and with $g = 32.2$, f/s², this makes $w = 0.0763$, lb/ft³, $C = 13.1$, ft³/lb, so that this standard air bulks 13 cubic feet to the pound, in round numbers.

Another way of expressing the law is to write it in the equivalent form

$$\frac{R}{S} = \left(\frac{V}{H} \right)^2, \text{ lb/ft}^2,$$

so that H is the velocity at which normal resistance is 1 lb per square foot; then on the figures above $H = 20.5$ f/s, and this may be replaced by 20 in round numbers for practical calculation, making

$$\frac{R}{S} = 25 \left(\frac{V}{100} \right)^2.$$

Flying over the ocean the velocity would be expressed in knots, K , and with 12 knots the equivalent of 20 f/s the formula is

$$\frac{R}{S} = \left(\frac{V}{20} \right)^2 = \left(\frac{K}{12} \right)^2, \text{ lb/ft}^2;$$

simple numbers easily remembered.

In all these calculations Perry's dictum must be respected: that the accuracy of a formula is only the accuracy of its most inaccurate part.

Here the index 2 of the velocity, adopted for simplicity of calculation, is the part most subject to doubt, and then at this rate of the quadratic law the above numbers, 23.7, 24, and 25, are all equally