

known skill of the author, who has added to our libraries a most useful and interesting work. Both he and the Wireless Press, which has produced the book, may be cordially congratulated on the result of their labours.

Fermat's Last Theorem.

Three Lectures on Fermat's Last Theorem.

By L. J. Mordell. Pp. vii + 31. (Cambridge: At the University Press, 1921.) 4s. net.

THE "last theorem of Fermat" states that if x, y, z, p denote positive integers, the equation $x^p + y^p = z^p$ is impossible if p exceeds 2: thus no cube can be the sum of two cubes, and so on. If the theorem is true when p is 4, or an odd prime, it is true for all other integral values of p . For three centuries this theorem has baffled the efforts of all who have attacked it, although it has attracted the attention of all first-rate arithmeticians, and a great number of amateurs. For $p=3, 4, 5, 7$ comparatively simple proofs have been discovered; but so far none of these has led to a complete generalisation.

The first great advance in the theory was made by Kummer, in connection with his researches on cyclotomic integers. He showed that if the theorem is false for any particular odd prime p , then p must not be a factor of the numerator of any one of the first $\frac{1}{2}(p-3)$ numbers of Bernoulli. This very recondite test rules out all values of p below 100 except 37, 59, 67. By additional criteria Kummer was able to prove the theorem for these exceptional primes, and hence for all values of p from 3 to 100 inclusive.

Not many years ago (1907) a prize of 100,000 marks was set aside for the first who succeeded in giving a complete proof or disproof of the theorem. Quite recently, new criteria, independent of Kummer's, have been discovered, which have to be satisfied by odd primes p for which the theorem is false, and the simplest of these is the condition $2^{p-1} \equiv 1 \pmod{p^2}$, discovered by Wilferich in 1909. Other tests of a more or less similar kind have been accumulated, and the net result is that any value of p for which the theorem is false must exceed 7000. Gauss's tables of quadratic forms warn us not to draw any conclusions from this result; in fact if N is any assigned integer, however large, a proof that the theorem is true unless $p > N$ gives us no information about the truth or falsity of the theorem in general.

Mr. Mordell's lectures give a clear and interesting account of the history and present state of this subject. Lecture I. gives a statement of the

theorem, and a summary of the work done by Kummer's predecessors; Lecture II. is on Kummer's researches, and more recent investigations of similar type; and Lecture III. gives an account of various results obtained by Libri, Sophie Germain, and others. Full references are given to the original papers, so that a reader within reach of a good reference library can make himself acquainted with details of all that has been done hitherto.

A perplexing circumstance, often alluded to, is the fact that, in a private note, Fermat distinctly asserted that he had proved the theorem. Now Fermat was never convicted of a false assertion, and only once of a wrong conjecture; on the other hand it is extremely improbable that Fermat's proof, if he had one, was in any way analogous to the work of Kummer and his successors. It is not, perhaps, unreasonable to hope that a proof may be found, some day, derived from Diophantine analysis proper, combined with a process of induction, and possibly with some application of analytical geometry, or theory of equations, or both. A really gifted youth, approaching the problem without knowledge of modern analysis, might throw a quite new and unexpected light upon it.

Mr. Mordell's pamphlet ought to do much to stimulate our rising mathematicians, and we hope that it will have a large circulation.

G. B. M.

Chemistry of Coke-oven and By-product Works.

Coke-oven and By-product Works Chemistry.

By T. Biddulph-Smith. Pp. x + 180 + 7 plates. (London: Charles Griffin and Co., Ltd., 1921.) 21s.

THE author states in the preface that his object in compiling this book is to furnish a concise manual covering, so far as space will allow, the general work required for the chemical control of coke-oven and by-product works. As regards the variety of subjects treated, he has doubtless achieved his object, but it is to be regretted that the apparent exigencies of space have caused the manual to become so concise in certain sections as to detract appreciably from the value of the work as a whole.

The most valuable section of the manual is that relating to the coal-tar naphthas. There is no doubt that the author has taken considerable pains to collect together the work of some of our best analytical chemists on methods of evaluating the constituents of coal-tar naphthas—work which