Letters to the Editor.

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Phenomena of "Intelligence" in the Protozoa.

I REGRET to observe the spirit of the letter in which Mr. Dunkerly (NATURE, May 26, p. 395) replies to Mr. Ludford, though, as being primarily responsible for the statement of the theory suggested, and periodically referred to of late years, I fully realise how extremely careful one should be in the choice of words in conducting the discussion. There is probably no theory occupying the attention of zoologists in connection with which the motto of the Royal Society, "Nullius in verba," applies with greater force. Unfortunately, the "journalistic instinct" of many writers on scientific subjects has led them to credit observers with views which they have—to put it mildly—not yet reached, and to saddle them with responsibilities which they have never assumed. For instance, in Prof. Boycott's letter on the same page he credits my friend Earland with my views on "the selective intelligence of the Foraminifera," which is the one subject upon which my esteemed collaborator does *not* entirely agree with me.

The term "gregarious instinct" used by Mr. Ludford is an unfortunate one. The "grouping" of Protozoa to which he refers must be considered with a cautious appreciation of the elements of (*a*) fear, (*b*) reflex action, and (*c*) surface tension, but the most indignant opponent of my views will scarcely deny that the sense of fear is perhaps the most elementary phenomenon dependent upon a sensory system. It is, no doubt, related to, but it must not be confounded with, the "intelligence" displayed by many arenaceous Foraminifera in building their tests of *adventitious* material, and in using that material in such a manner as to protect the surface of the test from naturally incidental dangers of damage, and to protect the apertures of the tests against the entrance of predatory parasites.

The "grouping" to which Mr. Ludford directs attention must not be confounded with the associations of marine Rhizopoda, which gain protection against suffocation in soft muds by the co-operative use of spicules, arranged as catamaran spars to maintain them upon the surface (as in *Psammosphaera rustica*, H.-A. and E.), or with the aggregation of simple arenaceous tests for purposes of strength and protection, which, unfortunately, has led some of the earlier rhizopodists to treat such associations as new genera or species. It is as if they were to describe a litter of little pigs huddled together for warmth (which is an elementary phenomenon of intelligence) as a new and "polythalamous" genus of pig.

EDWARD HERON-ALLEN.

Large Acres, Selsey, May 31.

An Algebraical Identity $4X=Y^2-37Z^2$.

THE following is a well-known theorem derived from the theory of numbers. Let p be any ordinary odd prime, and let $X=(x^p-1)/(x-1)$; then there is an algebraical identity

$$4X=Y^2\pm pZ^2,$$

where Y, Z are polynomials of degree $\frac{1}{2}(p-1)$ and $\frac{1}{2}(p-3)$ respectively; and the sign of the ambiguity is + or − according as p is of the form $4n+3$ or $4n+1$. The cases up to $p=31$ inclusive have been published; the result for $p=37$ has just been communicated to me by Pundit Oudh Upadhyaya, research scholar of the University of Calcutta. He finds that

$$4X=Y^2-37Z^2$$

with

$$Y=2x^{18}+x^{17}+10x^{16}-4x^{15}+15x^{14}-5x^{13}+17x^{12}-8x^{11}+11x^{10}-4x^9+11x^8-8x^7+17x^6-5x^5+15x^4-4x^3+10x^2+x+2.$$

$$Z=x^{17}+0x^{16}+2x^{15}-x^{14}+3x^{13}-x^{12}+2x^{11}-x^{10}+2x^9-x^8+2x^7-x^6+3x^5-x^4+2x^3+0x^2+x.$$

I have tested this result in various ways, and have no reason to doubt its correctness.

It should be noted that Y may be obtained by expanding $2(x-1)^{18}$, and reducing the coefficients to their absolutely least residues mod. 37. It would be interesting to know the least value of p for which this rule does not apply. It must be less than 61.

G. B. MATHEWS.

7 Menai View, Bangor, May 29.

Atmospheric Refraction.

THE following proposition regarding the effects of refraction may be known, but I do not remember to have seen it stated. It is: "The course of a nearly horizontal ray of light in the lower part of the atmosphere is a circular arc having a radius of 14,900 geographical miles."

The velocity of light in that lower part of the atmosphere for which the decrease of pressure with the increase of height is nearly linear is given by the relation

$$v_h=v_0\left(1-a\frac{H-h}{H}\right),$$

where v_0 is the velocity *in vacuo*, v_h the velocity at the height h above the ground, and H the height of

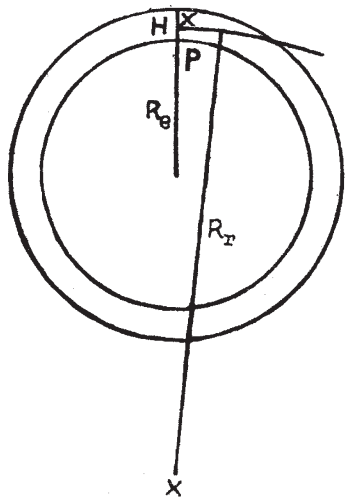

FIG. 1.

the homogeneous atmosphere ($a=00029$ nearly). At ground level the velocity is $v_0(1-a)$.

Let a plane vertical wave surface start from P as in Fig. 1. After the lapse of the time t it will have advanced v_0t at the height H, and $v_0(1-a)t$ at the surface of the ground. (This assumes the linearity of the relations between v_h and h to hold up to H, and though this is not true, the conclusions drawn from the assumption are correct, at any rate up to a few thousand feet.)

Thus at the time t the wave surface will be inclined forward, making an angle

$$\frac{v_0-v_0(1-a)}{H}t, \text{ or } v_0t\frac{a}{H},$$