Mercury's apsidal motion. For the expression $3\left(\frac{\text{velocity of planet}}{\text{velocity of light}}\right)^2$, which is the angular motion of the apse (in fractions of a circumference) per revolution of the planet, involves no empirical or arbitrary constant. We can express the reason for the advance in simple terms thus: At infinity the relative velocity is zero, and the law is the Newtonian one, but the nearer we approach the central orb the higher becomes the velocity, and the greater the extra force. Hence we have another case of the force falling off more rapidly than the inverse square, which we have seen to lead to apsidal advance.

It is interesting to note that the advance per revolution varies as (velocity)^2 or as $1/a$. Hence the advance per century varies as $a^{-\frac{5}{2}}$, or it falls off much more rapidly with increase of a than the Hall law, which gives $a^{-\frac{3}{2}}$. In the course of centuries this would discriminate between them, independently of the lunar test; but the orbits of Venus and the earth are so nearly circular that the time for that test has not yet arrived.

In the case of Mars we may note that F. E. Ross's rediscussion of the observations of that planet and of the mass of Venus takes off some 2″ from Newcomb's value of its excess of apsidal motion in a century. When we further remove the Einstein term 1·3″, we are left with some 2·7″; as the actually observed quantity is the product of 2·7″ by the eccentricity (1/11), it falls well within the limits of observational error.

The Displacement of Solar Lines.

By DR. CHARLES E. ST. JOHN, Mount Wilson Solar Observatory, Pasadena, California.

THE agreement of the observed advance of Mercury's perihelion and of the eclipse results of the British expeditions of 1919 with the deductions from the Einstein law of gravitation gives an increased importance to observations on the displacement of absorption lines in the solar spectrum relative to terrestrial sources, as the evidence on this deduction from the Einstein theory is at present contradictory. Particular interest, moreover, attaches to such observations, inasmuch as the mathematical physicists are not in agreement as to the validity of this deduction, and solar observations must eventually furnish the criterion.

Prof. Eddington's view, if I understand it, is that the theory cannot claim support from the present evidence, nor can observed displacements not agreeing with the theory be on that account regarded as in the slightest degree adverse to it, the only possible conclusion in his view being that there are certain causes of displacement of the lines acting in the solar atmosphere and not yet identified ("Space, Time, and Gravitation," p. 130).

The great majority of metallic lines observed for differences between their wave-lengths in the sun and terrestrial sources do show displacements. These differ in most cases from those deduced from the Einstein law of gravitation in ways as yet unexplained. If reasonable solar causes can be adduced to account for the wide discrepancies between theory and observation, the position of the generalised theory of relativity would be greatly strengthened.

According to the theory, the displacements are to the red, and are proportional to wave-length, being independent of the intensity of the lines and of the element to which they belong. Observational results differ from those deduced from the theory in at least four important ways. The observed displacements are not proportional to the wave-length; they differ from element to element in the same spectral region; for the same element and spectral region they vary with line-intensity; the displacements show both large positive and negative divergences from the calculated values. Interesting examples are found in Jewell's early work (*Astrophysical Journal*, vol. iii., p. 89, 1896). The relative values here are of high weight, and the data are important in that the range of elements is wider than occurs in more recent investigations. Divergences in the four directions from the calculated displacements are shown in the following extract from his observations on the differences in wave-length between 115 solar and arc lines:—

			Mean λ	Δ λ Obs.	Δ λ Cal.	Obs. – Cal.
Calcium	H,K,	4227	4042	+0·019	+0·009	+0·010Å
Calcium	Int.	1–6	5227	+0·004	+0·011	–0.007
Iron	,,	10–40	3950	+0·008	+0·008	±0·000
Iron	,,	2–8	3950	+0·003	+0·008	–0·005
Aluminium	,,	15–20	3950	+0·006	+0·008	–0·002
Nickel	,,	10–15	3530	+0·017	+0·008	+0·009
Nickel	,,	1–5	3625	+0·005	+0·008	–0·003
Copper	,,	6–9	3262	+0·006	+0·007	+0·009
Potassium	,,	00–0	4046	–0·008	+0·009	–0·017

For statistical discussion the quantity of data available is as yet quite inadequate even in the case of iron, the most widely studied element. Not only should the terrestrial and solar wave-lengths be known to high precision over the widest possible range of spectrum, but also the pressure shift per atmosphere. Unfortunately, there are no published data on the wave-lengths and pressure displacements of the iron lines, in which, over a long spectral range, the errors due to pole-effect in the arc are reduced to the magnitude of the calculated Einstein displacement. For other metallic elements the data are even more deficient. With a sufficiently large and varied accumulation of material there is hope that the complex solar conditions may be analysed, and the contributions to the observed effects arising from the various causes determined with some certainty. The pressing need is for data of the requisite accuracy and variety. This need adds interest to determinations of wave-lengths and of pressure displacements, and to investigations of the characteristic behaviour of spectrum lines, as all such data will have a part in solving one of the most absorbing questions in cosmic physics.

Evershed adduces his observations upon the spectrum of Venus as evidence of an "earth-effect" driving the gases from the earth-facing hemisphere of the sun, and he would by this hypothetical action explain the observed displacements of the solar lines, and thus negative the deduction from the Einstein theory. Two series of Venus observations have been made by Dr. S. B. Nicholson and myself. The details will appear in a forthcoming Contribution from the Mount Wilson Observatory. Our observations indicate that the displacements of the Venus lines to the violet relative to skylight are due to non-uniform illumination of the slit when the guiding is done upon the visual image, the effect increasing with the refraction and becoming more evident the smaller the image. The explanation is based upon the observation that spectrograms taken at low altitudes give larger displacements to the violet than those taken on the same night at higher altitudes, and that the displacements correlate with the cotangent of the altitude and the reciprocal of the diameter of the planet at the time of observation.

In respect to the observations at Mount Wilson on the lines of the cyanogen band at $\lambda 3883$, I have as yet found no grounds for considering them seriously in error. The explanation of the results adverse to the theory based upon dissymmetry appears inadequate (*Observatory*, p. 260, July, 1920), and the assumption that the adverse results are due to superposed metallic lines seems to be negatived by the observations of Adams, Grebe, Bachem, and myself that for these lines there is no displacement between the centre and limb of the sun. Metallic lines as a class shift to the red in passing from the centre to the limb. If, then, metallic lines are superposed on these band lines in such a way as to mask the gravitational displacement to the red when observed at the centre of the sun, this should be revealed by a shift to the red at the limb.

The lines of the cyanogen bands are under investigation in the observatory laboratory both as reversed in the furnace and as produced in the arc under varying pressure. The measures show no evidence of a displacement to the red under decreased pressure as indicated by Perot's observations.

The present programme at Mount Wilson aims at an accumulation of varied and extensive data that will furnish a suitable basis from which to approach the general question of the behaviour of Fraunhofer lines relative to terrestrial sources.

Non-Euclidean Geometries.

By PROF. G. B. MATHEWS, F.R.S.

THE ordinary theory of analytical geometry may be extended by analogy as follows: Let $x_1, x_2, \ldots x_n$ be independent variables, each ranging over the complete real (or ordinary complex) continuum. Any particular set $(x_1, x_2, \ldots x_n)$, in that order, is said to be a point, the co-ordinates of which are these x_i; and the aggregate of these points is said to form a point-space of n dimensions (P_n). Taking $r<n$, a set of r equations $\phi_1=0, \phi_2=0, \ldots \phi_r=0$, connecting the co-ordinates, will in general define a space P_{n-r} contained in P_n. Theorems about loci, contact, envelopes, and the principle of duality all hold good for this enlarged domain, and we also have a system of projective geometry analogous to the ordinary one.

Physicists are predominantly interested in metrical geometry. The ordinary metrical formulæ for a P_3 may be extended by analogy to a P_n; there is no logical difficulty, but there is, of course, the psychological fact that our experience (so far) does not enable us to "visualise" a set of rectangular axes for a P_n if $n>3$; not, at least, in any way obviously analogous to the cases $n=2, 3$.

In ordinary geometry, for a P_3 we have the formula

$$ds^2=dx_1^2+dx_2^2+dx_3^2$$

for the linear element called the distance between two points (x), $(x+dx)$. Riemann asked himself the question whether, for every P_n, this was necessarily a typical formula for ds, on the assumption that solid bodies can be moved about in space without distortion of any kind. His result is that we may take as the typical form, referred to orthogonal axes,

$$ds^2=\Sigma dx^2/N^2,$$

where $$N=1+\tfrac{1}{4}\alpha\Sigma x^2,$$

and α is an arbitrary constant, called the *curvature* of the P_n in question. This curvature is an intrinsic property of the P_n, and should not be considered as a warp or strain of any kind. When $\alpha=0$, we have the Euclidean case. As an illustration of the theory that can be actually realised, take the sphere $x^2+y^2+z^2=r^2$ in the ordinary Euclidean P_3. By putting

$$D=u^2+v^2+4r^2,$$
$$Dx,\ Dy,\ Dz=4r^2u,\ 4r^2v,\ (u^2+v^2-4r^2)r,$$

the equation $x^2+y^2+z^2=r^2$ becomes an identity, and we may regard the surface of the sphere as a P_2 with (u, v) as co-ordinates. The reader will easily verify that

$$ds^2=(du^2+dv^2)\div\{1+\frac{1}{4r^2}\ (u^2+v^2)\}^2;$$

so we have a case of Riemann's formula with $\alpha=r^{-2}$. We *cannot* find a similar formula for the surface of an ellipsoid, because a lamina that "fits" a certain part of the ellipsoid cannot be