duce colour, though no finer structure can anywhere be made out, section 9 (Fig. 2).

Euploea deione.—There is one large group of insects of considerable interest, the colour of which cannot be accounted for in any way. This group includes the dark purples and deep, glossy blues and greens of all the most sombre iridescent insects, such as the Purple Emperor (*Apatura iris*), the Scarlet Tiger (*Callimorpha dominula*), the Purple Hairstreak (*Zephyrus quercus*), and many exotic species. Any one of these, such as the section of *Euploea deione*, *11* (Fig. 2), shows no difference from the black, non-iridescent scales immediately beneath them. They are all so densely pigmented that nothing can be made out until they are bleached, and even then a thin cuticle only sometimes appears. Were it not for the fact that the colours disappear under pressure and in refractive fluids, it might be thought that the iridescence was due to selective metallic reflection, as will be shown to be probably the case in most scaleless beetles.

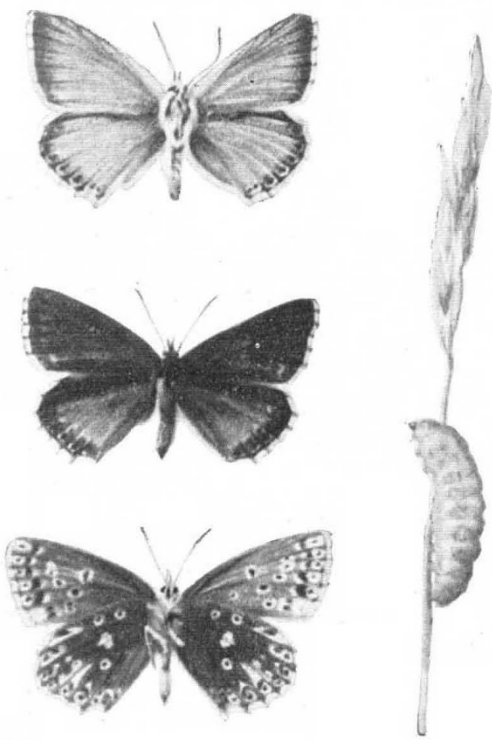

FIG. 6 *Lycaena corydon* (the Chalk Hill Blue). Male, female, under-side of female and larva. (Natural size.)

Dione juno.—Scarcely less puzzling are the metallic greenish-gold and silver scales of many Plusia, such as the Burnished Brass Moth (*Plusia chrysitis*). Section *8*, Fig. 2, shows the golden scale of the tropical insect *Dione juno,* which has no structure that can adequately account for the colour, since it is identical with the scales in the adjoining brown areas, except for the absence of the pigment. To produce anything approaching metallic reflection a highly polished surface would be necessary, as well as a large number of air-spaces not more than the diameter of a few air-molecules in thickness. The effect of a highly polished surface is seen in the scales of the Coppers, as, for instance, the Small Copper (*Chrysophanus phlaeas*), which has ordinary scales containing a granular orange pigment, yet appearing almost iridescent. The only trustworthy evidence of true iridescence is, of course, the change of colour seen on altering the angle of the incident light.

(*To be continued.*)

Ballistic Calculations.

By D. R. HARTREE.

THE purpose of the present article is to give an outline of the more important methods of numerical solution of the various problems of external ballistics—that is to say, problems connected with the resisted motion of a shell after leaving a gun. Most of the methods to be mentioned were developed during the war, either for working out range tables or other information to be used in the field, or for analysing a trial shoot.

The problems that arise may conveniently be divided into two groups, comprising what are sometimes known as primary and secondary problems, the theoretical and practical treatments of which proceed on rather different lines. The primary problems are those which involve the calculation of the performance of a gun, or rather its shell, under ideal conditions, such as still air, a standard muzzle velocity, and so on. The secondary problems are those in which we are concerned with the calculation of the corrections to be applied to the solutions of the primary problems to allow for the departure of actual conditions from the ideal.

A very important simplification is introduced by assuming that the forces due to the motion of the shell through the air consist only of a resistance in a direction opposite to the direction of the motion of the shell relative to the air. Lack of trustworthy information until recently about the other forces made this the only possible course. Some account of these forces and their effects is given in a recent issue of NATURE.[1]

Making this assumption, and neglecting the effect of earth rotation (which may be considered as a secondary problem), it appears that the trajectories concerned in any primary problem lie entirely in the vertical plane containing the initial direction of motion. For this reason they are known as "plane trajectories."

The retardation due to air resistance R acting

[1] See NATURE, June 10, "The Dynamics of Shell Flight," by R. H. Fowler.

on a shell of mass m is usually expressed by the formula:—

$$\frac{R}{m}=\frac{F(v)}{Cf(y)},$$

where v is the velocity of the shell relative to the air, $f(y)$ is the reciprocal of the density of the air at height y, $F(v)$ is an experimentally determined function of the velocity, and C depends on the size and shape of the shell.

Standard functions $F(v)$ and $f(y)$ are used in all ordinary calculations; the quantity C is determined for any particular shell by comparing the results of firing trials with trajectories calculated for the same muzzle velocity and elevations as used in the trial, and two or three values of C.

Primary Ballistic Problems.

Before the advent of the anti-aircraft gun the point of fall was the only point of a trajectory of any great practical importance, and this could be found to a certain degree of accuracy by means of approximate integrals of the equations of motion, for high-velocity guns at small elevations, and for low-velocity howitzers at high elevations, which were the two cases of importance before the war.

But when guns began to be used at higher elevations, and the muzzle velocities of howitzers were increased, these approximate solutions became unsatisfactory; also, with the development of the anti-aircraft gun, came the necessity for calculating whole trajectories, instead of merely a point on each trajectory. Later, it was found necessary to know the whole trajectory, even for guns only intended for use against targets on the ground, in order to solve certain secondary problems, such, for example, as the wind correction to be applied when the wind varied with the height.

The equations of motion, even of the plane trajectory, are formally insoluble, which is not surprising, considering that the air resistance which enters into them contains two functions, $F(v)$ and $f(y)$, of an empirical nature. The only really satisfactory way of obtaining numerical solutions is to carry out a numerical integration of the equations of motion.

To perform this integration a step-by-step method is employed. That is to say, the trajectory is divided up into a series of fairly short intervals, and the integration through each interval in turn performed by means of suitable approximate formulæ, the size of the interval being chosen to make the errors negligible. The complicated way in which the different variables are connected makes it impossible to use directly any of the ordinary integration formulæ, such as Simpson's rule.

Methods of step-by-step integration have, of course, long been known in astronomy; they seem, however, to have been regarded until recently as too laborious for ballistic work except in special cases. However, during the war those concerned with ballistic calculation were forced to use them, for reasons already mentioned, and gradually with experience methods were evolved which were both simple to carry out and not too lengthy. The use of a series of intervals of the same length, and of the finite differences of various quantities at the ends of successive intervals, both simplifies the integration and makes possible a complete check on the numerical work.

When two or more trajectories of the same gun with different elevations have been calculated by these methods, it is obviously possible to determine intermediate trajectories by interpolation. Theoretically, interpolation methods are of a subsidiary nature; in practice, if simple and accurate, they are often very useful.

For a range table, or for the graduation of sighting apparatus, either for flat or high-angle fire, interpolation from the data furnished by the calculation of trajectories is necessary.

Thus in a flat range table the elevation necessary to reach a given range is tabulated as a function of the range, but in calculating a trajectory an exact value of the elevation is taken, and the range is found. The interpolation in this case is usually done graphically.

For a high-angle range table the question is more complicated, for this table is one of double entry, giving the elevation required to reach various points in a two-dimensional region. A table obtained by graphical interpolation usually needs some smoothing. This process, though fairly simple for a single entry table, is almost prohibitive for a table of double entry. A scheme of accurate numerical interpolation was therefore evolved; this scheme as a whole is rather elaborate, but the individual calculations are very simple.

In England the greater part of the numerical work of ballistics is carried out by means of calculating machines.

Secondary Ballistic Problems.

The development of the methods of solution of the secondary problems in general arose in the first place from the necessity of finding the effect of a wind, or change of atmospheric density from standard, which varied along the trajectory. These are the most important secondary problems in practice, but the methods can be extended to others with little difficulty.

To make the problem more manageable, only "first order" effects of applied variations are considered. That is to say, it is assumed that the effects of such variations are additive, so that, for example, the effect of a given wind and a given change of atmospheric density acting together is the sum of the effects of each separately. In cases of practical importance the error is probably very small.

The problem of calculating the effect of a wind variable along the trajectory is generally divided into two parts, the determination of the effect of unit constant wind, and the determination of the "equivalent constant wind"—*i.e.* the constant wind which produces the same effect at the same time. The latter is obtained by means of a set of "weighting factors" which express the relative importance of the wind at different parts of the trajectory.

Considering the wind effect at any given point, the weighting factor for any section of a trajectory is the ratio of the effect at that point of unit wind blowing in that section only to the effect at the same point of unit wind blowing throughout the trajectory. If W is the actual wind in any section, and k is the weighting factor for that section, then the equivalent constant wind is given by the sum of the values of kW for all sections up to the point where the effect of the wind is being considered.

The same arguments apply to variations of atmospheric density from standard. An account of the application of weighting factors has appeared in a recent number of NATURE[2]; we are concerned here with the calculation of them.

The values of the weighting factors for given sections depend on the point at which the effect of a wind (or change of density) is being calculated, and on the precise effect which is being considered. For example, a wind in the plane of the trajectory produces changes in both horizontal and vertical co-ordinates of the point reached in a given time, and if the wind varies along the trajectory the constant wind which will produce the same horizontal displacement will not generally produce the same vertical displacement.

The four equations of motion of the plane trajectory express the relations between the components of velocity, the co-ordinates, and the time. From them can be obtained, by a process analogous to differentiation, four "equations of variation" expressing the relations between the changes in these quantities, for a given time, for any change in conditions which causes first order variations in the plane of the trajectory. (A cross-wind produces only second order effects in this plane, and its treatment is entirely separate.)

The equations of variation have no formal solution, and step-by-step integration is necessary for numerical work. To find wind weighting factors, the obvious method is to integrate the equations for winds blowing in the sections for which weighting factors are required; but this is not necessary, for if the integration is performed for three suitable changes of conditions, the results may be combined to give weighting factors, not only for wind, but for density changes as well.

The numerical work of the process of combining the three solutions is rather heavy and not altogether simple, and a more direct way of calculating weighting factors has been worked out. The equations of variation form a system of linear differential equations of the first order, and by using a certain property of such a system another set of equations (the "adjoint" system) can be obtained, the solutions of which give directly the effect at a given time of a constant wind (or density changes) which begins at a previous variable time. Weighting factors are obtained at once by dividing by the effect of a constant wind which begins at the origin, and differencing the results.

The equations of variation may be applied to any problem in first order variations. The subject of second order variations has not been developed, as its practical importance appears small.

[2] NATURE, June 17, "The Importance of Meteorology in Gunnery," by Dr. E. M. Wedderburn.

Obituary.

THE study of earthquakes in New Zealand and Australia has suffered a serious loss through the death of MR. GEORGE HOGBEN on April 20 last. For many years Mr. Hogben acted as secretary of the seismological committee of the Australasian Association for the Advancement of Science, and we are indebted to him for reports of this committee, and for many studies of individual earthquakes published in the Transactions of the New Zealand Institute and other journals. It was owing to his efforts that the Milne seismograph was erected at Wellington, N.Z., and that, shortly before his death, an order was given for the improved Milne-Shaw seismograph. In addition to his contributions to our knowledge of earthquakes, Mr. Hogben was interested in education generally, and was for two years president of the Wellington Philosophical Society. According to a notice issued with the Hector Observatory Bulletin (No. 28, 1920), he also issued a valuable report on proportional representation, and at the time of his death had an improved set of mathematical tables in the press.

THE *Atti dei Lincei* (vol. xxix. (1), parts 9–10) contains an obituary notice by R. Versari on the late PROF. FRANCESCO TODARO. Born at Tripi (Messina) on February 14, 1839, Todaro entered the University of Messina in 1860, but on the entry of Garibaldi he took up arms as a volunteer in the Chasseurs of Etna. On the conclusion of hostilities and of service to the wounded, he returned to the University, and was attracted by the German biologists to anatomical and physiological studies. He went for some time to study at Florence under Schiff and others, and in 1865 published his first paper on the muscular system of the human heart and the Eustachian valve. He returned to Messina as professor of human anatomy, and in 1869 gave an address on the renewal of the human body. Todaro was among the earliest to study the anatomy of the lower marine animals, and to realise, in accordance with the doctrine of evolution, the importance of comparative anatomy as throwing light on the anatomy of man. In 1870 he read a paper on the sense-tubes of Plagiostomata, and the following year was invited by Brioschi to the chair