

$$a + 0.5000b + 0.3333c + 0.2500d + 0.2000g + 0.1667h + 0.1429j + 0.1250k + 0.1111l + 0.1000m,$$

that is, with

$$a + \frac{1}{2}b + \frac{1}{3}c + \frac{1}{4}d + \frac{1}{5}e + \frac{1}{6}f + \frac{1}{7}g + \frac{1}{8}h + \frac{1}{9}i + \frac{1}{10}m,$$

which is  $\int_0^1 f(x) dx$ .

An approximate evaluation of  $\int_0^1 f(x) dx$  is therefore given by

$$\frac{1}{4} [F(\frac{1}{10}) + F(\frac{4}{10}) + F(\frac{6}{10}) + F(\frac{9}{10})].$$

2. The following table shows for several functions the value of the integral and the approximate evaluation by this four-ordinate rule and by two seven-ordinate rules in common use, viz. :—

Simpson's rule :—

$$\int_0^1 f(x) dx = \frac{1}{8} [F(\frac{0}{8}) + F(\frac{2}{8}) + 2\{F(\frac{4}{8}) + F(\frac{6}{8})\} + 4\{F(\frac{1}{8}) + F(\frac{3}{8}) + F(\frac{5}{8}) + F(\frac{7}{8})\}], \text{ approx.}$$

Weddle's rule :—

$$\int_0^1 f(x) dx = \frac{1}{20} [F(\frac{0}{10}) + 5F(\frac{1}{10}) + F(\frac{2}{10}) + 6F(\frac{3}{10}) + F(\frac{4}{10}) + 5F(\frac{5}{10}) + F(\frac{6}{10})], \text{ approx.}$$

F(x)	$\int_0^1 f(x) dx$	New rule	Simpson	Weddle
Semicircle (x - x <sup>2</sup> ) <sup>½</sup>	$\frac{\pi}{8} = 0.3927$	0.3949	0.3815	0.3835
Quadrant (1 - x <sup>2</sup> ) <sup>½</sup>	$\frac{\pi}{4} = 0.7854$	0.7868	0.7775	0.7789
(4x - x <sup>2</sup> ) <sup>½</sup>	$\frac{2\pi}{3} - \frac{\sqrt{3}}{2} = 1.228$	1.231	1.217	1.219
log(1+x)	$2 \log 2 - 1 = 0.3863$	0.3859	0.3863	0.3863
$\frac{1}{e^x}$	$e - 1 = 1.718$	1.720	1.718	1.718
$\frac{1}{1+x}$	$\log 2 = 0.6931$	0.6937	0.6932	0.6931
$\frac{1}{2+x}$	$\log \frac{3}{2} = 0.4055$	0.4056	0.4055	0.4055
sin x	$1 - \cos \frac{180^\circ}{\pi} = 0.4597$	0.4593	0.4597	0.4597

3. The approximation is convenient for the practical determination of the area of a closed curve, such as an indicator diagram. The arithmetical mean of the ordinates at one-tenth, four-tenths, six-tenths, and nine-tenths of the range is the mean ordinate for the range.

The decimal division of the range, the use of only four ordinates, the extremely simple arithmetic involved, and the degree of accuracy attained should make the rule of practical value.

A. F. DUFTON.

Trinity College, Cambridge,  
April 30.

**British and Metric Systems of Weights and Measures.**

ARE not those who discuss the relative claims of 4 mills and 5 mils as the substitute for the penny in a decimal division of the pound merely trying to minimise the disadvantages of what must in any case be a change for the worse? It seems that the advantage of any given system of weights or measures lies largely in the facilities that it offers for the division of a sum or quantity into equal parts. In this respect

any decimal system is deficient by the absence of the factor 3, and by the frequency of the factor 5, which is of much less use than 4 for practical purposes. The *reductio ad absurdum* of the metric system seemed to be reached in the issue in Portugal some years ago of a 2½ reis postage stamp (they now call it ½-cent). A rei is one-thousandth part of a milrei or dollar, about equal to one-twentieth of a penny—surely a small enough unit for any purpose, and yet it is found necessary to halve it!

The following comparison seems instructive :—

No. of farthings in one pound = 960 = 2<sup>6</sup> × 3 × 5.

This has 11 factors between 1 and 20,  
20 factors between 1 and 100.

No. of inches in one mile = 63,360 = 2<sup>7</sup> × 3<sup>2</sup> × 5 × 11.

This has 14 factors between 1 and 20,  
34 factors between 1 and 100.

No. of ounces in one ton = 35,840 = 2<sup>10</sup> × 5 × 7.

This has 9 factors between 1 and 20,  
17 factors between 1 and 100.

No. of grains in one lb. troy = 5760 = 2<sup>7</sup> × 3<sup>2</sup> × 5.

This has 13 factors between 1 and 20,  
26 factors between 1 and 100.

No. of seconds in one day = 86,400 = 2<sup>7</sup> × 3<sup>3</sup> × 5<sup>2</sup>.

This has 13 factors between 1 and 20,  
32 factors between 1 and 100.

Contrast with these :—

No. of millimetres in one kilometre, or of grammes in one metric tonne = 1,000,000 = 2<sup>6</sup> × 5<sup>6</sup>,

which has only 7 factors between 1 and 20,  
14 factors between 1 and 100.

If all the above five English systems be taken together, it will be found that :—

The factor 2 occurs 37 times  
" " 4 " 17 "  
" " 8 " 11 "  
The factors 3, 6, and 12 occur 8 "  
" " 5, 10, 16, and 20 " 6 "  
The factor 15 occurs 5 "  
The factors 9 and 18 occur 3 "  
And the factors 7, 11, and 14 " once each.

Now, though it cannot be contended that the man who wants to divide 100l. into seven parts is helped by the fact that there are 28 lb. in a quarter, or he who would divide a ton into eleven parts by the number of yards in a furlong, yet it seems worthy of note that in our admittedly heterogeneous system all the numbers below 20, except 13, 17, and 19, should be represented as factors, and that to an extent so nearly proportional to their probable utility.

M. E. YEATMAN.

Parliament Mansions, May 7.

**Scientific Apparatus and Laboratory Fittings.**

I AM surprised to see that Prof. W. M. Bayliss, who writes in NATURE of May 6 on the proposed Anti-Dumping Bill, has misunderstood the Bill so far as it relates to scientific instruments. This Bill does not propose a tariff, but prohibition, except under licence.

The British Optical Instrument Manufacturers' Association has urged the Government to act by prohibition except under licence rather than by tariff, and this is what the Bill proposes. It has always considered that the effect of a tariff might, as Prof. Bayliss suggests, give "no inducement to the makers to improve the quality"; and it has urged that licences should always be freely granted where articles were not being made in the required quantity or up to the standard of quality of goods that could be imported from abroad.