Letters to the Editor.

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Gravitational Deflection of High-speed Particles.

MR. LEIGH PAGE has given a very simple method of treating the motion of high-speed particles in a gravitational field on Einstein's theory (NATURE, February 26, p. 692). In one respect his results differ from those which have been obtained by more laborious methods, and I think that some error must have crept in, either through a failure of his approximation or from some other cause. He finds that a particle travelling with the velocity of light would be undeflected, whereas a ray of light is deflected. It would be difficult to reconcile this with the principle of equivalence, which seems to require that the trajectory of a material particle should approach that of a light-pulse as the velocity approaches that of light.

The differential equation of the orbit of a material particle moving with any speed is [Report, Physical Society, p. 51, equation (31·2)]

$$d^2u/d\theta^2+u=m/h^2+3mu^2, \qquad (u=1/r),$$

where the constant $h=r^2d\theta/ds$. It is from this exact equation that the motion of perihelion of Mercury is obtained. For motion with the speed of light $ds=0$, so that h is infinite, and the equation becomes

$$d^2u/d\theta^2+u=3mu^2.$$

The solution is

$$u=\frac{\cos\theta}{R}+\frac{m}{R^2}\left(\cos^2\theta+2\sin^2\theta\right),$$

neglecting m^2/R^2.

In Cartesian co-ordinates this becomes

$$x=R-\frac{m}{R}\frac{x^2+2y^2}{\sqrt{(x^2+y^2)}}.$$

The asymptotes are found by taking y very large compared with x, giving

$$x=R\pm\frac{2m}{R}y.$$

Hence the angle between them is $4m/R$, agreeing with the result for the deflection of light rays.

I have verified by the usual methods the other principal result given by Mr. Page, that for radial motion the force (relative to the co-ordinates used) is a repulsion if the speed exceeds $1/\sqrt{3}$ times the velocity of light.

With regard to the question whether the system of an atom on the sun can be identical with that of an atom on the earth, inasmuch as the warping of space-time is different in the two places, it is clear that the identity cannot be *exact*; but this loophole for escape from the predicted shift of the Fraunhofer lines does not seem to be very promising. If the "intervals" of vibration of the two atoms are not the same, the difference must depend on some invariant of space-time which differs at the two places. I do not think that any invariant of order m/r exists. The simplest invariant which does not vanish is

$$g^{\mu\alpha}g^{\nu\beta}g^{\sigma\gamma}g_{\rho\delta}B^{\rho}_{\mu\nu\sigma}B^{\delta}_{\alpha\beta\gamma};$$

it is rather laborious to work out the actual value of this (since it consists of 65,536 terms), but it appears to be of order m^2/r^2. The Fraunhofer displacement depends on terms of the much greater order of magnitude m/r.

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Gravitational Shift of Spectral Lines.

THE assumption that the equations of motion in a gravitational field can be deduced from a condition of the form $\delta\int ds=0$ is in itself little more than a very natural way of expressing the principle of least action. The greatness of Einstein's theory really lies in the suggestion, made apparently on purely *a priori* grounds, that a certain set of six relations between the coefficients in the formula for ds^2, which are true when no heavy body is near, still hold near one. These are found to make the coefficients determinate, whereas previously they were quite arbitrary, and the observed motions of the planets, including the advance of the perihelion of Mercury, are at once deduced.

The displacement of star images during an eclipse is based on the further very plausible assumption that a light-wave moves like a material particle of zero mass starting from an infinite distance with the velocity of light there. Now that this displacement has become a result of observation, the data are just enough to make it possible to reverse the argument and deduce the fundamental assumption of the theory from observation, as I have done in a forthcoming paper in the Monthly Notices of the Royal Astronomical Society. Neither in Einstein's discussion nor in mine is any identification of ds with an invariable line element in four-dimensional space-time relevant to the theory; and as the application of the theory is purely physical, I think it undesirable that any such abstract idea should be made to appear as part of it. Physically, the invariance of ds means simply that the motion of a particle can be described in terms of any set of co-ordinates we like to choose.

In discussing these phenomena all positions and times are referred to an observer at the centre of the sun, and it is not necessary to determine the relations between his measures and ours, for the uncertainty in these would not affect the observed quantities appreciably. The problem of the shift of spectral lines, however, depends essentially on such a comparison. About part of the theory of it there can be no reasonable doubt, namely, the assumption that the vibration on the earth appears to any observer to have the same period as the vibration on the sun that causes it. What is doubtful is whether the atom on the sun vibrates in the same time as a similar atom on the earth. Einstein assumes that it does not, but that the increase in ds in a period is the same for both, and deduces the shift of the spectral lines.

There is nothing very bizarre about this; it only means that when we move about we must refer our observations to time standards in the place where these were originally used, and not expect that they will serve the same functions if we carry them about with us. An analogy from colour will illustrate this. Suppose we have a standard of redness in the form of a particular red body. We judge the redness of other bodies by comparison with this. Now suppose we go to a place where the prevailing illumination is green, but where our standard of redness is still visible through a window. We then say that none of the things in the room look red, but our judgments as to what outside bodies look red are the same as before. Our standard is now brought into the room. Are we going to say that it looks red still? If we do, we shall have to say that the red external bodies that have not been moved have been changed in colour by the motion of our standard, which is at least inconvenient, and which most people would call absurd. Therefore we say that our colour standard has been altered by its displacement, and choose another standard from among the visible external bodies.

Similarly, if an observer on the earth went to the