comes to $2 \cdot 3 \times 10^{-4}$ $\partial p/\partial x$ radians, where p is the surface pressure in dynes cm.$^{-2}$ and dx is in cm.

LEWIS F. RICHARDSON.

Benson Observatory, Wallingford,
December 12.

I, OF COURSE, admit the force of the remarks in the letters which appeared in NATURE of December 11. But the problem of air refraction during a total eclipse is a very complicated one. The air is not in equilibrium. There is, I imagine, a downward rush of cold air in places deprived of the sun's radiation, as well as a lateral motion of the air from all sides towards such places. The whole refraction effect depends on the shape of the changing surfaces of equal density, and the gradient of density perpendicular to these surfaces. The effect observed would be about equal to the ordinary refraction effect caused by the atmosphere at $1\frac{1}{2}°$ from the zenith, and then the rays of light are nearly perpendicular to the surfaces of equal density.

It is well to remember that, perhaps unfortunately, the stars in the neighbourhood of the sun during a total eclipse must be viewed through air of which the distribution of density must not be assumed to be the same as that of the atmosphere in its normal state.

ALEXR. ANDERSON.

EINSTEIN'S RELATIVITY THEORY OF GRAVITATION.[1]

III.—THE CRUCIAL PHENOMENA.

IN the article last week an attempt was made to indicate the attitude of the complete relativist to the laws which must be obeyed by gravitational matter. The present article deals with particular conclusions.

As Minkowski remarked in reference to Einstein's early restricted principle of relativity: "From henceforth, space by itself and time by itself do not exist; there remains only a blend of the two" ("Raum und Zeit," 1908). In this four-dimensional world that portrays all history let (x_1, x_2, x_3, x_4) be a set of co-ordinates. Any particular set of values attached to these co-ordinates marks an event. If an observer notes two events at neighbouring places at slightly different times, the corresponding points of the four-dimensional map have co-ordinates slightly differing one from the other. Let the differences be called (dx_1, dx_2, dx_3, dx_4). Einstein's fundamental hypothesis is this: there exists a set of quantities g_{rs} such that

$$g_{11}dx_1^2 + 2g_{12}dx_1dx_2 + \ . \ . \ . \ + g_{44}dx_4^2$$

has the same value, no matter how the four-dimensional map is strained. In any strain g_{rs} is, of course, changed, as are also the differences dx.*

If the above expression be denoted by $(ds)^2$, ds may conveniently be called the *interval* between two events (not, of course, in the sense of time interval). In the case of a field in which there is no gravitation at all, if dx_4 is taken to be dt, it is supposed that ds^2 reduces to the expression $dx_1^2 + dx_2^2 + dx_3^2 - c^2dt^2$, where c is the velocity of light. If this is put equal to zero, it simply expresses the condition that the neighbouring events correspond to two events in the history of a point travelling with the velocity of light.

Einstein is now able to write down differential equations connecting the quantities g_{rs} with the co-ordinates (x_1, x_2, x_3, x_4), which are in complete accord with the requirement of complete relativity.[2] These equations are assumed to hold at all points of space unoccupied by matter, and they constitute Einstein's law of gravitation.

Planetary Motion.

The next step is to find a solution of the equations when there is just one point in space at which matter is supposed to exist, one point which is a singularity of the solution. This can be effected completely[3]: that is, a unique expression is obtained for the interval between two neighbouring events in the gravitational field of a single mass. This mass is now taken to be the sun.

It is next assumed that in the four-dimensional map (which, by the way, has now a bad twist in it, that cannot be strained out, all along the line of points corresponding to the positions of the sun at every instant of time) the path of a particle moving under the gravitation of the sun will be the most direct line between any two points on it, in the sense that the sum of all the intervals corresponding to all the elements of its path is the least possible.[4] Thus the equations of motion are written down. The result is this:

The motion of a particle differs only from that given by the Newtonian theory by the presence of an additional acceleration towards the sun equal to three times the mass of the sun (in gravitational units) multiplied by the square of the angular velocity of the planet about the sun.

In the case of the planet Mercury, this new acceleration is of the order of 10^{-8} times the Newtonian acceleration. Thus up to this order of accuracy Einstein's theory actually arrives at Newton's laws: surely no dethronement of Newton.

The effect of the additional acceleration can easily be expressed as a perturbation of the Newtonian elliptic orbit of the planet. It leads to the result that the major axis of the orbit must rotate in the plane of the orbit at the rate of $42 \cdot 9''$ per century.

Now it has long been known that the perihelion of Mercury does actually rotate at the rate of about $40''$ per century, and Newtonian theory has never succeeded in explaining this, except by *ad hoc* assumptions of disturbing matter not otherwise known.

Thus Einstein's theory almost exactly accounts

1 Previous articles appeared in NATURE of December 4 and 11.

* The gravitational field is specified by the set of quantities g_{rs}. When the gravitational field is small, these are all zero, except for g_{44}, which is approximately the ordinary Newtonian gravitational potential.

2 These equations take the place of the old Laplace equation $\nabla^2 V = 0$. Just as that equation is the only differential equation of the second order which is entirely independent of any change of ordinary space co-ordinates, so Einstein equations are uniquely determined by the condition of relativity.

3 The result is that the invariant interval ds is given by

$$ds^2 = (1 - 2m/r)(dt^2 - dr^2) - r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

the four co-ordinates being now interpreted as time and ordinary spherical polar co-ordinates.

4 This corresponds to the fact that in a field where there is no acceleration at all the path of a particle is the shortest distance between two points.

for the one outstanding failure of Newton's scheme, and, we may note, does not introduce any discrepancy where hitherto there was agreement.

The Deflection of Light by Gravitation.

The new theory having justified itself so far, it was thought worth while for British astronomers to devote their main energies at the recent solar eclipse to testing its prediction of an entirely new phenomenon.

As was remarked above, the propagation of light in the ordinary case of freedom from gravitational effect is represented by the equation $ds=0$.

This Einstein boldly transfer to his generalised theory. After all, it is quite a natural assumption. The propagation of light is a purely objective phenomenon. The emission of a disturbance from one point at one moment, and its arrival at another point at another moment, are events distinct and independent of the existence of an observer. Any law that connects them must be one which is independent of the map the observer uses; ds being an invariant quantity, $ds=0$ expresses such an invariant law.

This leads at once to a law of variation of the velocity of light in the gravitational field of the sun.

$$v=c(1-2m/r).$$

Here m, as before, is the mass of the sun in gravitational units, and is equal to 1·47 kilometres, while c is the velocity of light at a great distance from the sun. Thus the path of a ray is the same as that if, on the ordinary view, it were travelling in a medium the refractive index of which was $(1-2m/r)^{-1}$. In this medium the refractive index would increase in approaching the sun, so that the rays would be bent round towards the sun in passing through it. The total amount of the deflection for a ray which just grazes the sun's surface works out to be 1·75″, falling off as the inverse of the distance of nearest approach.

The apparent position of a star near to the sun is thus further from the sun's centre than the true position. On the photographic plate in the actual observations made by the Eclipse Expedition the displacement of the star image is of the order of a thousandth of an inch. The measurements show without doubt such a displacement. The stars observed were, of course, not exactly at the edge of the sun's disc; but on reduction, allowing for the variation inversely as the distance, they give for the bend of a ray just grazing the sun the value 1·98″, with a probable error of 6 per cent., in the case of the Sobral expedition, and of 1·64″ in the Principe expedition.

The agreement with the theory is close enough, but, of course, alternative possible causes of the shift have to be considered. Naturally, the suggestion of an actual refracting atmosphere surrounding the sun has been made. The existence of this, however, seems to be negatived by the fact that an atmosphere sufficiently dense to produce the refraction in question would extinguish the light altogether, as the rays would have to travel a million miles or so through it. The second suggestion, made by Prof. Anderson in NATURE of December 4, that the observed displacement might be due to a refraction of the ray in travelling through the earth's atmosphere in consequence of a temperature gradient within the shadow cone of the moon, seems also to be negatived. Prof. Eddington estimates that it would require a change of temperature of about 20° C. per minute at the observing station to produce the observed effect. Certainly no such temperature change as this has ever been noted; and, in fact, in Principe, at which the Cambridge expedition made its observations, there was practically no fall of temperature.

Gravitation and the Solar Spectrum.

It was suggested by Einstein that a further consequence of his theory would be an apparent discrepancy of period between the vibrations of an atom in the intense gravitational field of the sun and the vibrations of a similar atom in the much weaker field of the earth. This is arrived at thus. An observer would not be able to infer the intensity of the gravitational field in which he was placed from any observations of atomic vibrations in the same field: that is, an observer on the sun would estimate the period of vibration of an atom there to be the same that he would find for a similar atom in the earth's field if he transported himself thither. But on transferring himself he automatically changes his scale of time; in the new scale of time the solar atom vibrates differently, and, therefore, is not synchronous with the terrestrial atom.

Observations of the solar spectrum so far are adverse to the existence of such an effect. What, then, is to be said? Is the theory wrong at this point? If so, it must be given up, in spite of its extraordinary success in respect of the other two phenomena.

Sir Joseph Larmor, however, is of opinion that Einstein's theory itself does not in reality predict the displacement at all. The present writer shares his opinion. Imagine, in fact, two identical atoms originally at a great distance from both sun and earth. They have the same period. Let an observer A accompany one of these into the gravitational field of the sun, and an observer B accompany the other into the field of the earth. In consequence of A and B having moved into different gravitational fields, they make different changes in their scales of time, so that actually the solar observer A will find a different period for the solar atom from that which B, on the earth, attributes to his atom. It is only when the two observers choose so to measure space and time that they consider themselves to be in identical gravitational fields that they will estimate the periods of the atoms alike. This is exactly what would happen if B transferred himself to the same position as A. Thus, though an important point remains to be cleared up, it cannot be said that it is one which at present weighs against Einstein's theory.

E. CUNNINGHAM.