opinion on this matter, and this may have led certain of the contributors to hesitate in giving it. Others, however, have been able to read more clearly the signs and portents of the times, and, on the whole, their testimony is reassuring and full of hope. There can be no doubt whatever that the general body of chemical manufacturers in this country, as well as of the manufacturers dependent on chemical industry, have had a rude awakening. The war has completely upset commercial conditions, and many generations must come and go and a long period of peace ensue before pre-war relations are resumed. Public sentiment will force this country to depend more and more upon its own efforts, and to develop to a far greater extent its own internal resources. There is a general recognition that at the base of this problem is our educational system, and we see the evidence of this fact in the appointment of a professed educationist as director of a new policy. It is being realised that science and the methods of science must enter more largely into the curriculum of our secondary schools, and that colleges of science must be multiplied and strengthened. It is now everywhere perceived that the future of all industries depends upon science and upon the application of scientific principles. The bread that has been cast upon the waters is now being found after many days.

Many proofs of this fact are to be met with in the volume before us, accompanied, we regret to add, with certain disquieting features. There are those who aim at ends which are not those of their country, and too many new activities are secret. Perhaps in the circumstances this is unavoidable; but, as the example of our enemies has shown us, those industries flourish best and develop most rapidly where their leaders cooperate for their common good, even though they may themselves combine *contra mundum*.

Progress in applied chemistry may be measured by different standards. From an economic point of view it may be estimated by the wealth it brings to a community. This aspect of the matter finds practically no mention in the compilation before us. It is probably difficult to get together the requisite information, but if the Society of Chemical Industry could be induced to add a statistical department to its staff and publish the results of its labours each year as a supplement to these annual reports, we should obtain a real and valuable measure of the progress of the chemical arts in this country. As it is, the present work is too obviously based upon the pattern of the annual reports published by the Chemical Society, and is too exclusively a *catalogue raisonné* of the yearly output of the literature of applied chemistry. We would by no means undervalue the worth of such a compilation, but we venture to believe a fuller measure of its usefulness might be secured by a further extension of its scope.

These observations are offered in no spirit of carping criticism. We welcome with sincere pleasure the advent of an enterprise which is bound to have a far-reaching influence on the development of chemical industry in all English-speaking countries. Its inception at the present juncture is most timely, and we heartily wish it success. Thanks to the energy, skill, and perspicacity with which it is conducted, the journal of the society has become its most valuable asset. We are confident that these annual reports are destined to be a no less valuable feature of its work, provided that those who control its affairs are determined to rise to the full extent of their opportunity.

THE RADIATION OF THE STARS.

SINCE the publication of Homer Lane's paper "On the Theoretical Temperature of the Sun" in 1870, many writers have discussed the internal state of a star, considered as a globe of gas in equilibrium under its own gravitation. Recent observational work gives encouragement to these investigations, for it is now known that numerous stars are in a truly gaseous condition with mean densities similar to that of our atmosphere. To such stars the results for a perfect gas may fairly be applied, whereas stars, such as the sun, with densities greater than water must necessarily deviate widely from the theoretical conditions. The stars which are in a perfectly gaseous state correspond to the "giants" on H. N. Russell's theory,[1] or to the stars of rising temperature on Lockyer's principle of classification; the denser "dwarfs" are outside the scope of this discussion. The two series coalesce for spectral type B, which marks the highest temperature attained.

The internal temperatures which have been calculated are so far beyond practical experience that we may well hesitate to apply the familiar laws of physics to such conditions. But in so far as the investigation can be based on the second law of thermodynamics, the conservation of momentum, or laws which are directly deduced from these, there can be little doubt of the validity of the treatment. We cannot altogether avoid assumptions of a speculative or approximate character, and no doubt some of the results described in this article are open to serious criticism on that account; but to a considerable extent the discussion can be made to rest on laws which are held to be of universal application. Moreover, natural phenomena usually become simpler at high temperatures; gases become more "perfect"; the absorption of X-rays follows simpler laws than the absorption of light; the heat-energy comes to be located in greater proportion in the ether, so that the precise nature of the material atoms is less important.

Most investigators have assumed that the stars are in convective equilibrium.[2] In that case, when

[1] NATURE, vol. xciii., pp. 227, 252, and 281.

[2] There are strong reasons for believing that the interior of a star must be in radiative equilibrium, not convective equilibrium. The internal distribution of temperature and density is, however, of the same character in either case; if the coefficient of absorption is independent of the temperature, then the distribution corresponding to radiative equilibrium is the same as that of material for which $\gamma = \frac{4}{3}$ in convective equilibrium. See *Monthly Notices, R.A.S.*, vol. lxxvii., p. 16.

the mass and mean density are given, and also the molecular weight and ratio of specific heats (γ) of the material, we can find at once the temperature at any internal point. Let us take a star of mass $1\frac{1}{2}$ times that of the sun and of mean density 0·002 gm./cm.^{3}; for illustration, the average molecular weight will be taken as 54 (*e.g.* iron vapour dissociated into atoms at the high temperature). For γ we shall take $\frac{4}{3}$, but any possible change in γ makes comparatively little difference in the results, so far as we require them. For this star the calculated temperature at the centre is 150,000,000°; half-way from the centre to the boundary it is 42,000,000°. But the temperature of which we have some observational knowledge is not given immediately by these calculations; according to observation, the "effective temperature" of a star of this density would probably be about 6500°. This term does not refer to the temperature at any particular point, but measures the total outflow of heat per unit surface. Now, the outflow of heat evidently depends on two conditions—the temperature gradient (more strictly the gradient of T^4), and the transparency of the material; therefore, the temperature-distribution being calculated as already explained, we can deduce the transparency necessary to give the observed effective temperature of 6500°. The result is startling. We find the material must be so absorbent that a thickness of one-hundredth of a millimetre (at atmospheric density) would be almost perfectly opaque. There is little doubt that such opacity is impossible. Conversely, if we adopt any reasonable absorption coefficient, the effective temperature would have to be above 100,000°, which is decisively contradicted by observation.

A way out of this discrepancy is found if we take into account the effect of the pressure of radiation. Fortunately, this effect can be calculated rigorously without introducing any additional assumption or hypothesis. Suppose that a beam of radiation carrying energy E falls on a sheet of material which absorbs kE and transmits $(1-k)$E. It is known from the theory of electromagnetic waves that radiant energy E carries a forward-momentum E/c, where c is the velocity of light; similarly, the emergent beam carries momentum $(1-k)E/c$. The difference kE/c cannot be lost, and must evidently remain in the absorbing material. The material thus gains momentum, or, in other words, experiences a pressure. The amount of the pressure kE/c involves the coefficient of absorption k, of which we have no immediate observational knowledge; but it is the *same* coefficient which has already entered into the calculations of the opacity of the material, so that the introduction of radiation-pressure into the theory brings in no additional unknowns or arbitrary quantities.

The radiation-pressure is thus proportional to k, and to the approximately known outflow of energy. The preposterous value of k already found would, if adopted, lead to a pressure far exceeding gravity, so that the star would be blown to pieces. But the radiation-pressure modifies the internal distribution of pressure and temperature; it supports some of the weight of the outer layers of the star, and consequently a lower temperature will suffice to maintain the given density. The smaller temperature-gradient causes less tendency to outflow of heat, and there is accordingly no need for so high an opacity to oppose it. By calculation we find that for a star of mass 1·5 times the sun, and molecular weight 54, radiation-pressure will counterbalance 19/20ths of gravity; somewhat unexpectedly, this fraction depends neither on the density of the star (so long as it is a perfect gas) nor on the effective temperature, but it alters a little with the mass of the star. The pressures and temperatures are then reduced throughout in the ratio 1/20; for the star already considered, the corrected value of the central temperature is 7,000,000°. Assuming an effective temperature of 6500°, we can now calculate the new value of k; it amounts to 30 C.G.S. units, *i.e.* 1/30 gm. per sq. cm. section will reduce the radiation passing through it in the ratio $1/e$. It is of considerable interest to note that this is of the same order of magnitude as the absorption of X-rays by solid material; for at the high temperatures here concerned the radiation would be of very short wave-length and of the nature of soft X-rays.

The approximate balance between radiation-pressure and gravity leads to an important relation between stellar temperatures and densities. It is easy to put this relation in a more rigorous form; but it will suffice here to express the condition as radiation-pressure = gravity. If T is the effective temperature of the star, and g the value of gravity at the surface, the outflow of radiation (per unit area) varies as T^4, and the condition is

$$kT^4 \propto g.$$

We shall assume that k is the same for all stars. Now g depends on the mass and mean density in the ratio $M\,\rho^{\frac{2}{3}}$. Hence

$$T \propto M^{\frac{1}{12}}\rho^{\frac{1}{6}}.$$

The range of mass in different stars is trifling compared with the great range of density. Thus the leading result is that *the effective temperature of a giant star is proportional to the sixth-root of the density.* To test this, we take the densities given by Russell[3] for the different types, and, assuming that stars of the solar type (G) have the sun's effective temperature (6000°), we calculate by the sixth-root law the temperatures of the other types.

Type		Density (☉=1)		Effective temperature
A	...	$\frac{1}{10}$	...	10,800°
G	...	$\frac{1}{350}$	...	6,000°
K	...	$\frac{1}{2800}$	...	4,250°
M	...	$\frac{1}{25000}$	...	2,950°

The calculated numbers in the last column agree almost exactly with the temperatures usually

[3] *Loc. cit.*, pp. 282–83.

assigned to these types, and it is clear that if Russell's densities are correct the sixth-root law must be close to the truth.

If a is the radius of a star the total radiation will be proportional to a^2T^4, which varies as ga^2, *i.e.* as M. The total radiation thus depends only on the mass, and not on the density or stage of evolution. The absolute luminosity is a fairly good measure of the total radiation for the range of temperature here considered, though, of course, the visibility of the radiation changes a little with the temperature. We shall thus have the total radiation constant as we pass through the series of spectral types, and the luminosity roughly constant (with deviations amounting to about $1\frac{1}{2}$ magnitudes). This is just the feature which Russell has pointed out in the luminosities of the giant stars; they are practically the same whatever the type of spectrum.[4]

It may be remarked that this theory avoids a difficulty noticed by J. Perry[5], that when γ is less than $\frac{4}{3}$, the heat within the contracting star is greater than the energy set free by contraction, leaving less than nothing for radiation into space; the difficulty is even more serious than Perry considered, for he did not make any allowance for the enormous store of ethereal energy necessary for equilibrium with matter at high temperatures. But we have seen that by taking account of radiation-pressure the interior temperature is much reduced; less internal heat is therefore needed; and there is, in fact, an ample balance of energy left for dissipation even when γ is considerably below $\frac{4}{3}$.

With a molecular weight smaller than 54 the importance of radiation-pressure is reduced; for example, with molecular weight 18 radiation-pressure is 6/7 of gravity, instead of 19/20. But it still plays a predominant part until we come down to molecular weight 2. Reasons have been urged in favour of a low average molecular weight—perhaps as low as 2. It is probable that the atoms are highly ionised by the radiation of short wave-length within the star; and if most of the electrons outside the nucleus are split off from each atom we shall actually have an average weight for the ultimate independent particles nearly equal to 2, whatever the material (excluding hydrogen). Radiation-pressure is then less than half gravity; but the two principal laws, which seem to be verified by observation, are arrived at as before. Moreover, the order of magnitude of k is scarcely altered; it is now 5 instead of 30 C.G.S. units. Nor is the internal temperature much changed. In fact, the effect of ionising the atoms is that the pressure of the superincumbent layers is supported by a mixture of cathode rays and X-rays, instead of by X-rays alone; our doubt as to the proportions in which these occur and as to which will predominate is no serious hindrance, because the main results are nearly the same in any case.

A. S. EDDINGTON.

[4] *Loc. cit.*, p. 252, Figs. 1, 2, and 3.
[5] NATURE, vol. lx., p. 350.

DR. W. H. BESANT, F.R.S.

THE death of William Henry Besant on June 2, in his eighty-ninth year, will be mourned, in all sincerity, by a far greater number than he would have anticipated, supposing that he ever wasted a thought on the subject. Among these will be a legion of his old pupils, who had the opportunity of learning to know him in a peculiarly intimate way. Until 1880 or so Besant and Routh had almost a monopoly, for many years, in coaching pupils for the Mathematical Tripos. Besant's method was rather odd, but very effective with the right sort of man. At the cost of immense labour he had written out, with his own hand, a set of "bookwork and rider" papers covering the whole range of the examination. The pupil, on each of his three weekly visits, found one of these papers awaiting him in the outside room, and proceeded to answer it as well as he could on the backs of old examination scripts. If he had not brought a pen of his own, he had to search among a lot of ancient quills until he could find one that was not hopelessly spoiled. Presently, Mr. X would be politely summoned to an inner parlour, where his last exercise would be returned to him corrected and annotated, and if he had failed to answer any question he would be either shown a solution or given a hint how to proceed.

Of course, it was not every pupil that was taken separately like this; some of them were taken in small batches (not exceeding five or six), but the general method was the same. It should be added that once every week each pupil took away with him a printed problem paper to be done at leisure in his own rooms. The results were marked, and the list was available for inspection.

As a member of St. John's College staff Besant used to give "lectures" of a sort; but (unlike Routh) he eschewed formal lectures on bookwork. His solutions of problems were always original and elegant, and he had the great advantage (for a coach) of being equally good in geometry, analysis, and dynamics.

Besides being one of the *par nobile fratrum* of coaches, Besant was a busy and trusted examiner, and in this connection it may be recorded that he used to say that ten minutes of oral examination were worth any amount of written *ditto.*

Besant was too much engrossed by his proper work to add much to mathematical literature. His text-books on conics, dynamics, hydrostatics, and hydrodynamics deserved their popularity, and are still worth consulting, though their point of view is now rather antiquated. His one thoroughly original printed work, the tract on roulettes and glissettes (first edition, 1869; second edition, enlarged, 1890), shows all his qualifications at their best. Besant had really studied Newton, and had an exceptional power of estimating different orders of infinitesimals from a figure. His invention of the term