DIRICHLET SERIES.

The General Theory of Dirichlet Series. By G. H. Hardy and Marcel Riesz. Pp. x+78. (Cambridge: At the University Press, 1915.) Price 3*s.* 6*d.* net.

IT is well known that Dirichlet made a new start in the theory of numbers by bringing it into connection with certain analytical functions of the form $\sum_h n_h^{-s}$, n_h being an integer, and s a real quantity. In the present tract the definition of a Dirichlet series is

$$f(s)=\sum_1^{\infty} a_n \exp(-\lambda_n s),$$

where (λ_n) is a sequence of real increasing numbers converging to $+\infty$, and s is a complex variable. This obviously includes Dirichlet's functions, and also those considered by Dedekind, Riemann, Landau, and others, in the same connection.

In the earlier chapters we have a summary of the known elementary properties of $f(s)$, such as the conditions of its convergence when regarded as a sum in the ordinary sense, and so on. But the most novel, and principal, part, due mainly to Dr. Riesz, is concerned with what he calls the summability (λ, κ) of $f(s)$. It is important that the meaning of this should be properly understood. Let s_n mean the sum of the first n terms of $f(s)$; then Dr. Riesz associates with this a function which we may call $\sigma_n(\lambda, \kappa)$, where κ is an arbitrary positive number, and λ is a functional symbol defined by $f(s)$ and κ, so that we have a sequence $[\sigma_n(\lambda, \kappa)]$ which we may consider as depending upon κ. If this new sequence has a limit L, we say that $f(s)$ is summable (λ, κ). In the definition we only consider the *formal* nature of $f(s)$, so that it does not matter whether $f(s)$, regarded as a series in the ordinary way, is convergent or not. Thus we have a sort of extension of the theories of Poincaré, Borel, etc., about divergent series.

The conditions that L may exist give properties of $f(s)$ considered as a formal expression, and hence certain conclusions of an arithmetical nature can be drawn. It is in the further development of these deductions that we may hope for further information of interest. The mere fact that from a sequence (s_n) we may be able to construct a sequence (σ_n) which agrees, in the limit, with (s_n) when the latter is convergent, and may be called its "sum" when it becomes divergent in the ordinary sense, is a barren definition until we apply it to something concrete; if we can do this, it may be a very valuable resource, as in the case of asymptotic summation. Everything goes to show that the + we use in writing infinite series is in some ways less appropriate than the comma of the old-fashioned "progressions"; it will not matter if we remember what we are doing, and that there is no such thing as the actual sum of an infinite number of terms.

One observation in this tract is liable to misunderstanding. It may be true that Cahen is the first to discuss $f(s)$ *systematically* as a function of a complex variable s; but the first step in this direction was taken by Riemann in his famous paper on the distribution of primes. To arithmeticians, at any rate, it is the development of Riemann's results that is of principal interest at present; other problems of the same kind naturally present themselves, as, for example, the frequency of cases where p and $(p+2)$ are both primes, like (5, 7) or (11, 13). The new analysis may throw some light on these and other dark places. In any case, this tract will be welcome for its concise statement of known facts, and its bibliography, which supplements that of Landau.

G. B. M.

OUR BOOKSHELF.

Fungoid Diseases of Farm and Garden Crops. By Dr. T. Milburn. Pp. xi+118. (London: Longmans, Green and Co., Ltd., 1915.) Price 2*s.* net.

THIS book is intended "primarily for the use of farmers, gardeners, and agricultural students," and it is hoped that it may assist also "those engaged in teaching and county lecturing." The essentials of a book fulfilling these aims must be that for the student the elementary scientific facts are correctly and clearly stated, and for the farmer and gardener that his understanding of the subject is developed by his attention being directed to general principles while practical help is given. This book falls short of this standard.

The farmer and gardener fail to get the knowledge they want when they are told that "no exact formula" can be given for making Bordeaux mixture, and that "it is always necessary to test the solution before applying, for if too much lime be present it is useless as a fungicide." While the author quotes the titles of recent works on the chemistry of Bordeaux mixture, it seems scarcely possible that he has read them from the account he gives of the chemical composition of Bordeaux mixture. The statement that the preparation of soda-Bordeaux is "somewhat critical," and that since this mixture "possesses no effectual advantage" it can be passed over, will considerably surprise the Irish Department of Agriculture and the numberless farmers who, under their tuition, have saved thousands of acres of potatoes from "blight" by the use of this "soda-Bordeaux" or "Burgundy mixture." The gardener and fruit-grower will not acquiesce in the strange statement that "washes of potassium sulphide" are "very effectual, but too expensive."

The book is well printed and very cheap; it would have been better, however, to have in-