LETTERS TO THE EDITOR.

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The Principle of Similitude.

DR. RIABOUCHINSKY directs attention concisely in NATURE (July 29, p. 591) to an important point, which must have arrested the notice of readers of Lord Rayleigh's weighty exposition and illustration of the scope of the method of dimensions, as an instrument of precision in the analysis of physical problems. The example under consideration was the cooling of a hot wire by a stream of air passing across it. The point is that temperature, although in ultimate analysis it must be expressible in terms of the three fundamental dynamical entities—mass, space, and time—can yet be in that problem considered *effectively* as a fourth independent entity, thus vastly increasing the information derivable from comparison of dimensions. In the formal analysis of mere diffusion or conduction this is clearly valid, for the dynamical aspect of temperature is not involved.

In so far as thermodynamic considerations, such as work of expansion, etc., are of secondary importance in the analysis of the convection from a hot wire (as is the case so long as only an adhering surface layer of the gas need be taken as operative¹), the same principle will apply approximately there. But in a problem the content of which is mainly thermodynamic, the relations will be far more complex. In fact there is nothing transcendental about dimensions; the ultimate principle is precisely expressible (in Newton's terminology) as one of *similitude*, exact or approximate, to be tested by the rule that mere change in the magnitudes of the ordered scheme of units of measurement that is employed must not affect sensibly the forms of the equations that are the adequate expression of the underlying relations of the problem.

J. L.

Cambridge, July 31.

THE question raised by Dr. Riabouchinsky (NATURE, July 29, p. 591) belongs rather to the logic than to the use of the principle of similitude, with which I was mainly concerned (NATURE, March 18, p. 66). It would be well worthy of further discussion. The conclusion that I gave follows on the basis of the usual Fourier equations for conduction of heat, in which heat and temperature are regarded as *sui generis*. It would indeed be a paradox if the further knowledge of the nature of heat afforded by molecular theory put us in a worse position than before in dealing with a particular problem. The solution would seem to be that the Fourier equations embody something as to the nature of heat and temperature which is ignored in the alternative argument of Dr. Riabouchinsky.

August 2.

RAYLEIGH.

The Probable Error of the Amplitudes in a Fourier Series obtained from a Given Set of Observations.

To express a periodic variation by one or more terms of a Fourier series is such a common and convenient method that it is of importance to know the extent to which the constants are trustworthy. This is particularly the case when the quantity is

¹ Cf. also L. V. King, on hot-wire anemometry, *Phil. Trans.*, A 214 (1914), p. 373.

subject to a large casual and therefore non-periodic variation as well.

Suppose there are n evenly spaced observations, $y_0, y_1, y_2, \dots, y_{n-1}$, then confining attention to one term only, y may be expressed by the relation $y = a \sin(x - \alpha)$. This may be written

$$y = p \sin x + q \sin\left(x + \frac{\pi}{2}\right)$$

and the amplitude a is given by $a = \sqrt{p^2 + q^2}$.

The standard error of a therefore depends on that of p and q . To obtain p we write—

$$p = y_0 \sin 0 + y_1 \sin \frac{2\pi}{n} + y_2 \sin \frac{4\pi}{n} + \dots + y_{n-1} \sin \frac{n-1}{n} 2\pi$$

whence, if σ is the standard error of the y 's, it is easy to prove that $\sigma\sqrt{2}/\sqrt{n}$ is the standard error of the p or q . This is also the standard error for any higher order term depending on the n observations.

From the way in which p and q are obtained it is obvious they may be positive or negative; the sign depends, in fact, upon the origin chosen, and their mean value obtained from many samples would be zero. But even in the case of there being no period at all, and the y 's being purely casual, it is very improbable that both p and q should be zero. In this special case the amplitude a is really zero, but it is almost certain, since it equals $\sqrt{p^2 + q^2}$, not to be given as zero. Its mean square is plainly equal to twice the mean square of p or q , but this is not the square of the standard error, because it is not taken about the mean value.

To ascertain the general magnitude of the error the following plan has been adopted. From a sort of roulette board of 100 compartments, 500 pairs of numbers have been taken, the numbers on the board being arranged to have a standard deviation of 10, and a distribution as nearly normal as the limit of 100 numbers permits. From these pairs, representing the p and q of the sine curve, 500 amplitudes have been found. The result gives a mean amplitude of 12.5, a standard deviation of 6.5, and a distribution such that 4 per cent. exceed the value 25.

Similar results from 500 other pairs, but with a genuine sine curve of amplitude 10 superadded, give a mean amplitude of 16.5, a standard deviation of 8.0, and a distribution such that 8 per cent. are below 5, and 5 per cent. above 30.

From theoretical considerations it is apparent that as the periodic part of the variation becomes large compared with the casual part, the mean amplitude will still exceed, but will approximate to, the genuine amplitude, and the standard error will approximate towards that of p or q .

Suppose, then, that we have a set of n well-distributed observations, and that nothing is known about them in regard to their periodic variation. Let their standard deviation be σ . Now let the first three terms of the Fourier series be found in the usual way, and let the amplitudes be a_1, a_2 , and a_3 , the question that arises is, to what extent do a_1, a_2 , and a_3 represent genuine periodic variations? The standard deviation σ may be due entirely to the periodic variations, or may be purely casual. In the former case it is easily proved that $\sigma = \sqrt{a_1^2 + a_2^2 + a_3^2}/\sqrt{2}$, and if this relationship is approximately satisfied, the larger amplitudes may be accepted as correct. But if the deviation σ of the observations is distinctly larger than that produced by the periodic part, the casual part of it will lead to error in the a_1, a_2 , and a_3 , and unless an amplitude when compared with $\sigma\sqrt{2}/\sqrt{n}$ is well outside the limits for the special values given above, it need not be significant.