TWO MATHEMATICAL COURSES.

(1) *The Theory of Numbers.* By Prof. R. D. Carmichael. Pp. 94. (Mathematical Monographs, No. 13.) (New York: John Wiley and Sons, Inc.; London: Chapman and Hall, Ltd., 1914.) Price 4s. 6d. net.

(2) *Lectures Introductory to the Theory of Functions of Two Complex Variables.* Delivered to the University of Calcutta during January and February, 1913, by Prof. A. R. Forsyth. Pp. xvi+281. (Cambridge: University Press, 1914.) Price 10s. net.

THESE works are mainly interesting as examples of the trend of instruction given to university students who are just beginning to specialise. One is based upon two years' teaching in Indiana University; the other is the substance of a course delivered, by invitation, at the University of Calcutta, with, presumably, a number of Indian students in the audience. Both courses agree in the endeavour to trace the main outlines of the argument, to give really illustrative examples, and to avoid excessive detail on particular points of minor importance.

(1) Prof. Carmichael does not advance beyond well-beaten and classical ground, so that he has little opportunity of introducing really modern ideas. All but one of his six chapters deal with such things as ordinary factorisation of integers, elementary congruences, Fermat's theorem, and primitive roots. Chapter vi. is a miscellany in which there is a statement, without proof, of the law of quadratic reciprocity for two odd primers; a very brief account (with references, fortunately) of some special Galois imaginaries, and some interesting samples of the true Diophantine analysis. Some of the harder examples are novel and interesting, *e.g.* "The number of divisions to be effected in finding the G.C.M. of two numbers by the Euclidean algorithm does not exceed five times the number of digits in the smaller number (supposed written in the decimal scale)."

Without advising any improper and premature specialisation, it may be fairly urged that some of the work contained in this course might be done at school; the G.C.M. theory, and that of linear congruences, at any rate. Familiarity with the congruence notation is so important that its introduction ought not to be deferred.

(2) Prof. Forsyth has undertaken the difficult task of giving an outline of the known theory of functions of two independent (complex) variables, together with an account of what he calls "triple theta-functions." Everyone who has studied this theory at all is aware that it suggests theorems similar to those of Gauss, Green, Cauchy, etc., for functions of one variable, but that it is very difficult to prove them in an analogous, so to speak, geometrical way, mainly because while a plane or a sphere gives us a graphic picture of the field of one complex variable, we cannot at present realise a convenient image of the field of two independent complex variables. It is easy enough to make images of a sort; for instance, take two planes, plot off the independent variables, x, y, in the usual way on each, and now associate with a given pair (x', y'), the line joining the point which is the image of x' to that which is the image of y'. An obvious objection to this is that the meet of the planes does not correspond to a unique pair x', y', and there are other inconveniences connected with the difficulty of visualising linear complexes and congruences. After reviewing various proposals, Prof. Forsyth concludes that the only practicable way at present is the purely analytical one, following the methods of Weierstrass and his school. It is to be hoped that this is not the final word on the question; at any rate, there are papers by Picard, Appell, and Poincaré which ought to stimulate those whose ideas naturally clothe themselves in geometrical forms.

Features of the course which should be noted are: (1) the introduction of *two* dependent variables, now and then, as functions of two independent variables; the use of this is analogous to that of one-one transformations of plane curves; (2) in the chapter on integrals, where we have *two* algebraic functions introduced, so that the independent variables are arbitrary; (3) the theory of the so-called "triple theta-functions." The characteristic equations of these functions are given in the form:

$$\vartheta(z+1, z') = \vartheta(z, z'+1) = \vartheta(z, z')$$
$$\vartheta(z+\mu, z'+\mu') = \exp.\{-2\pi i(2z+2z') - 2\pi i(u+\mu')\}.\vartheta(z, z').$$

Hence the author derives double Fourier expansions for the functions, which fall into a co-ordinate set of sixteen; various tables and formulæ relating to them are given. The course concludes with a sketch of the theory of quadruply periodic (double) theta-functions, and their algebraic relations.

It seems to us that this course is not quite so well-proportioned or up-to-date as that of Prof. Osgood, recently published, on the same subject; but the difference of object and of audience must be allowed for. At any rate, we have a useful guide to the work of Weierstrass and Picard, a certain amount of new, although not very fundamental, theory; some instructive and original examples; and, it need scarcely be said, an elegant analytical presentation of the subjects treated.

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