

laboratory for the estimation of acidity in peats from different types of moors. The most satisfactory results were obtained by treatment of the peat with calcium acetate and estimation of the free acid resulting.

Dr. Tacke (Bremen) supported the method adopted at Bremen, *i.e.* neutralisation with  $\text{CaCO}_3$ , on the ground that it involves the actual practical method of getting rid of acidity in moors. Moreover, whether the acid properties are due to the presence of actual acids or to colloids,  $\text{CaCO}_3$  is the best and most commonly applied neutraliser.

Tacke's method found many supporters who frequently use it and have always found it efficient. Results obtained at Bremen with the two methods,  $\text{CaCO}_3$  and calcium acetate, agreed very well. It became at once noticeable that the former diversity of opinions between the stations at Bremen and Munich, as to the nature of soil acidity, no longer exists to the same extent. It appears to be generally accepted that the acidity of sour soils may be due to the presence of both actual acids and colloids. Tacke still maintains that the fact that most colloids present in soils are acid is sufficient to account for the views previously put forward by Baumann and Gully.

The discussion was chiefly remarkable for bringing out the large number of methods which have been employed by different workers. These included the direct method of obtaining an alkali extract, precipitating the brown colloidal matter with neutral calcium chloride, and titrating the clear solution. Another method was the estimation of the H ions in a water extract. It was considered that much more research is required before any particular method can be adopted officially.

A committee, consisting of Prof. Albert, Prof. Rindell, Dr. Tacke, and Dr. Gully was appointed to test thoroughly the different methods.

After the meeting the members of the commission were conducted by Prof. Kraus through an interesting collection of soils in his laboratory, including typical agricultural soils of Bavaria and other German States and also a large collection from the German colony of Togo; then by Prof. Henkel through the other laboratories of the agricultural section of the Munich Technical High School.

J. A. H.

THE STARS AROUND THE NORTH POLE.<sup>1</sup>

THIS is a mathematical problem which can be solved fairly easily, and the answer is that the stars must be distributed in distance according to a law shown graphically by the curve in Fig. IV. (The distribution of velocities  $\frac{h}{\sqrt{\pi}} e^{-h^2 v^2} dv$  combined with the distribution of proper motions  $\frac{dr}{a} \left(1 + \frac{r^2}{a^2}\right)^{-\frac{3}{2}}$  leads to the partial distribution  $2a^2 h^2 r e^{-a^2 h^2 r^2} dr$ ).

In the diagram, distances are measured horizontally, the unit of distance being that at which a star's parallax is equal to 1" (or 206,265 times the distance of the earth from the sun). It is convenient to have a name for this unit, and in what follows the word *Parsec*, suggested by Prof. Turner, will be adopted. With this unit a distance of 100 in the diagram denotes twenty million times the distance of the sun

from the earth. The following table gives the percentage of stars between certain limits of distance:—

TABLE IV.

6 per cent. of the stars are between 0 and 100 parsecs				
5	"	"	"	100 " 200 "
10	"	"	"	200 " 400 "
43	"	"	"	400 " 700 "
36	"	"	"	>700 "

It follows that 88 per cent. of the stars in Carrington's Catalogue—that is, 88 per cent. of all the stars brighter than about 10.5 magnitude—lie between 20 and 150 million times the distance of the sun from the earth. This law of the distribution of the stars is at first sight rather surprising. It should be remembered that the only stars at a great distance which are included are those which are intrinsically very bright, and these form only a small proportion of all the stars. Prof. Eddington has found that a similar law holds for stars brighter than 6.0 magnitude.

Having found the law of distribution of the distances of these stars, it is not difficult to determine something about their absolute luminosities, *i.e.* how they would compare with the sun in brightness if placed at an equal distance from us.

If the sun were at a distance of one parsec, it would appear as a bright star, brighter than the first

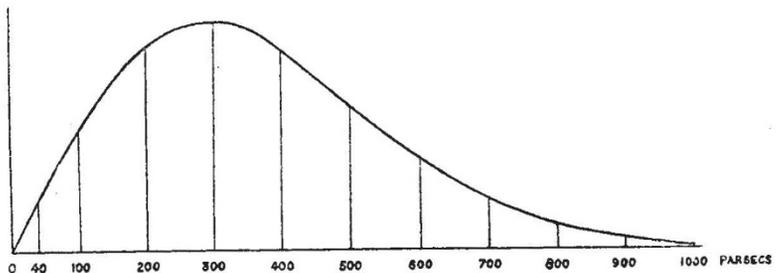


FIG. 4.—Distribution in distance of the stars in Carrington's Catalogue.

magnitude—actually of magnitude 0.5; if at a distance of 100 parsecs, its magnitude would be 10.5. Now all the stars in Carrington's Catalogue may be taken as brighter than 10.5 magnitude, thus at least 95 per cent. of these stars are intrinsically brighter than the sun, and at least 80 per cent. are four times as bright, 40 per cent. are sixteen times as bright, and 8 per cent. are fifty times as bright.

We may conclude that the great majority of the stars brighter than 10.5 magnitude are intrinsically brighter than the sun, and a considerable proportion very much brighter.

The distribution of bright and faint stars in a given volume of space is quite different, and contains a much larger proportion of faint stars. If we make the assumptions that the density of the stars and the proportions of bright and faint ones is the same at the different distances from the sun within which these Carrington stars are situated, it is possible to find the actual number of stars of different luminosities in a given volume of space. In a sphere with radius 100 parsecs, or twenty million times the distance of the earth from the sun, there are, at least,

24	which are	100	times as	luminous	as the	sun
340	"	50	"	"	"	"
1,530	"	25	"	"	"	"
4,840	"	10	"	"	"	"
23,200	"	1	"	"	"	"
93,300	"	$\frac{1}{10}$	th	the	luminosity	of the sun.

<sup>1</sup> Discourse delivered at the Royal Institution on Friday, April 24, by Dr. F. W. Dyson, F.R.S. Continued from p. 576.

The data only admit of a rough determination of the number of very faint stars and the number of very bright ones. The figures give a general indication of the density of the stars in space and of their intrinsic brightness, and serve to direct attention to the fact that there are many stars much less luminous than the sun, and a certain proportion very much more luminous.

The conclusions drawn up to this point have been based entirely on a consideration of the proper motions of the stars, irrespective of whether they are bright or faint, provided only that they are sufficiently bright to have been observed by Carrington. But as the apparent magnitude of a star depends on its distance as well as on its intrinsic brightness, we naturally expect some assistance in assigning the distances of these stars from their magnitudes. The brightest star in this small area round the North Pole is Polaris, the magnitude of which is 2. (It may be remarked incidentally that the distance of the pole star has been actually measured. It is twenty parsecs, or four million times the distance of the sun from the earth, and if it were at the same distance as the sun it would appear to be 100 times as bright.) Then there are about twenty stars which are visible to the naked eye. The following table gives the actual number of stars of different magnitudes (photographic):—

Magnitude	Number of stars
Brighter than 7 <sup>o</sup> m. ...	61 stars
From 7 <sup>o</sup> m. to 8 <sup>o</sup> m. ...	124 "
" 8 <sup>o</sup> m. " 9 <sup>o</sup> m. ...	397 "
" 9 <sup>o</sup> m. " 10 <sup>o</sup> m. ...	998 "
Fainter than 10 <sup>o</sup> m. ...	2140 "

Then, again, the stars may be divided into groups according to the physical characteristics revealed by the spectroscope. The researches of Kapteyn, Campbell, and others have shown—at any rate, for the brighter stars—remarkable relationships between the distances and velocities of the stars and the type of spectrum which they manifest. It is therefore desirable to examine the proper motions of stars of different spectral types separately. The spectra of many thousands of stars have been determined at Harvard College, under Prof. Pickering's direction, by Miss Cannon. The different classes are indicated in the Harvard classification by the letters B, A, F, G, K, M, with further subdivisions. The B stars are characterised by the presence of helium, the A stars by series of broad hydrogen lines. In the F stars the hydrogen lines are thinner, and fine metallic lines are shown. The G stars are very like the sun, full of metallic lines, and with broad lines due to calcium. In the K stars the two calcium lines are still broader, and there are many fine metallic lines. The M stars are characterised by broad absorption bands. This arrangement places the stars in the order of their temperatures; the B stars are the bluest and hottest, and the M stars the reddest and coolest. The character of the spectra of about 800 of the stars in Carrington's Catalogue is given by the Harvard observations.

For the fainter stars the spectra have not been determined, but they can be inferred in another way. As the blue stars are more active photographically than the red stars, if a red and a blue star have the same visual magnitude, the magnitudes estimated from the images on a photograph will differ considerably, and this difference is an index of the colour, and thus of the type of spectrum. Now the visual magnitudes of most of these faint stars have been very accurately determined at Potsdam by Messrs. Müller and Kron (and have been kindly communicated in manuscript), and the photographic magnitudes

have been determined at Greenwich. The differences have been taken between the photographic and visual magnitudes, and serve to classify the stars according to their temperature.

Separating the stars into two groups, those which are brighter than 9.5 magnitude on the Potsdam scale of magnitudes, and those which are fainter than 9.5 magnitude, and dividing each group into four classes according to the colour index, the parallax motion, i.e. the mean angular movement per century arising from the motion of the sun through space, is determined for each class. The results are exhibited in the following table:—

TABLE VI.

Spectral class	Colour index	Stars brighter than 9.5 <sup>m</sup> .		Stars fainter than 9.5 <sup>m</sup> .	
		Number	Parallax motion	Number	Parallax motion
K—M	>8	175	0.65	269	0.36
G—K	4 to 8	168	1.31	428	0.95
F—G	-1 to 4	264	2.58	959	1.53
A—F	< -1	240	1.97	460	1.28

In this table the red stars are on the top line; the third line consists of stars which are in the same stage of development as the sun; those in the second line are somewhat cooler and redder; those in the last line hotter and bluer. The last line includes a few, but only a few, B stars, as there are not many in this part of the sky. The quantities in the fourth and six columns of the table are a gauge of the distance of the stars to which they refer. It is only necessary to divide these into 337", which is the angle through which a star distant 1 parsec would have been displaced in the solar motion in one hundred years, to obtain the distances in parsecs. Thus the 240 stars belonging to types A—F, and brighter than 9.5 magnitude, are at an average distance of 170 parsecs.

The first point to notice is that parallax motions of stars fainter than 9.5 magnitude are always considerably less than the corresponding quantities for stars brighter than 9.5 magnitude. This is, of course, because the faint stars are, on the whole, further away. The average distance of stars of magnitude 10.0 is approximately 1.4 times as great as for a star of 8.0 magnitude.

The next point is the very great distance of the red stars. The 269 faint red stars are very nearly 1000 parsecs away, or 200 million times as distant as the sun. At this great distance the sun would appear as of magnitude 15.5, but these stars vary in magnitude from 9.5 to 11.0, and are therefore intrinsically from 250 to 63 times as bright as the sun. Now it happens that among the stars nearest to the sun the distances of which have been actually measured there are several red stars, and these are all very much fainter than the sun. It has been suggested by Prof. Russell and Prof. Hertzsprung independently that the red stars are of two distinct classes, which they call the giants and the dwarfs, and that, in accordance with Sir Norman Lockyer's views, the giant red stars are in an early stage of evolution, and are increasing in temperature; while the dwarf stars are at the other end of the series, and are growing colder and darker.

Leaving the red stars, it is seen that the stars the colour indexes of which lie between -1 and +4 are nearer to us than the groups on either side of them. These stars are those the spectra of which are of the types F and G in the Harvard notation, and are the stars most like the sun. The mean distances of these stars is only 130 parsecs for the stars brighter than 9.5 magnitude, and 215 parsecs for the stars fainter than 9.5 magnitude. At this distance the sun would be of magnitude 12.1. It

follows that these stars are, on the average, from two to eight times as bright as the sun. The A-F stars are a little, but not much, further away, the stars fainter than 9.5 magnitude being at an average distance of 263 parsecs. At this distance the sun would have a magnitude of 12.5, and these stars are from sixteen to four times as luminous as the sun.

It has been shown how the knowledge that the solar system is moving in a known direction with a velocity of 19.5 km. per second leads to a determination of the distances of groups of stars the angular movements of which are known. The hypothesis made is that in a number like one hundred or two hundred stars, the irregular angular movements due to the motions of the stars themselves neutralise one another on the average. But this is only the mean distance of the group, and some are much nearer and some much further. The distribution of the stars about this mean distance may be derived from the proper motions, if we know how the linear velocities are distributed. I shall apply this method to the group of stars which are like the sun in type of spectrum, and therefore, presumably, of like temperature and physical constitution.

Dividing these into three classes according to their magnitude, it is found that their parallactic motion due to the sun's movement, and their average motion in the perpendicular direction due to their own peculiar movements, are as follows:—

	No.	Parallactic motion	Av. cross motion	Ratio
All stars down to 11.0m.	1247	1.92	± 1.67	0.87
Stars brighter than 10.0m.	470	2.50	± 2.10	0.84
" " 9.0m.	148	3.34	± 2.90	0.87

In the last column is given the ratio of the average cross motion to the parallactic motion. The agreement of the numbers shows that the bright stars and the faint stars have the same average velocity. Taking the velocity of the sun as 19.5 km. a second, it follows that the average velocity of these stars in the direction perpendicular to the sun's motion is 13.7 km. a second.

We shall now make the assumption that some of these stars are moving faster than this velocity and some slower, just as errors of observation are distributed about a mean error. With a mean velocity of 13.7 km. a second, there will be in 1000 stars

231	with velocities	0 to	5 km/sec.
208	" "	5 "	10
175	" "	10 "	15
141	" "	15 "	20
163	" "	20 "	30
59	" "	30 "	40
18	" "	40 "	50
1	" "	> 50	

If now the observed proper motions are arranged, it is found that the number less than any value  $\tau$  can be represented satisfactorily by an algebraic formula  $N \frac{\tau}{(\tau^2 + a^2)^{3/2}}$ , where N is the total number of stars and a is the mean value of  $\tau$ . The following table shows the actual number of stars with proper motions between certain limits, compared with the number given by the formula:—

TABLE VII.

Limits of proper motion	No. of stars observed	No. given by formula	Difference
0" to 1" a century	427	429	-2
1 " 2 "	346	337	+9
2 " 4 "	324	332	-8
4 " 7 "	105	103	+2
7 " 10 "	25	22	+3
> 10 "	20	19	+1

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We may take it that the formula substantially represents the observed facts. With the proper motions distributed according to this formula, and the actual velocities distributed according to the law of errors, the distribution of the stars in distance can be determined, and it is found that these 1247 stars are distributed in space as shown in Table VIII.

TABLE VIII.—Number of Solar Stars (Types F and G) at Different Distances.

Distance (parsecs)	Out of total 1247 stars	Out of 470 stars brighter than 10.0m.	Out of 148 stars brighter than 9.0m.
< 100	121	76	40
100—200	298	161	65
200—300	332	136	34
300—400	254	68	8
400—500	146	23	1
500—600	65	5	
600—700	23		
> 700	5		

The most remarkable feature of this table is that 70 per cent. of the stars lie between the narrow limits of one hundred and four hundred parsecs.

I have treated the 470 stars which are brighter than 10.0 magnitude and the 148 brighter than 9.0 magnitude in a similar manner. The results are given in the third and fourth columns of Table VIII. Taking the differences, the distribution in distance of the 777 stars of magnitude 10.0—11.0 and of the 322 stars of 9.0—10.0 magnitude is found.

To compare the intrinsic magnitudes of the stars it is convenient to take limits of distance in geometrical progression with a common ratio 1.259 (log=0.1), e.g. 40, 50, 63, 79, 100, 126, etc., parsecs. These limits correspond to a change of half a magnitude in the intrinsic brightness of the stars which are of the same apparent brightness. Confining our attention to the stars of apparent magnitude 10.0 to 11.0, or, speaking broadly, stars of 10.5 magnitude, the limits 50—63 parsecs contain stars half a magnitude brighter, and distributed over twice the volume of those contained between the limits 40—50 parsecs.

If we may assume that the actual density of the stars is the same in all parts of the space with which we are dealing, we obtain by reasoning of this kind the number of stars between different limits of absolute brightness. The following table shows the number of stars of different luminosities in a sphere of one hundred parsecs radius:—

Luminosity $\odot = 1$	No. of stars	
	10.0m.—11.0m.	9.0m.—10.0m.
0.40 to 1.0	16,000	18,000
1.0 " 2.5	9,500	11,200
2.5 " 6.3	5,750	7,300
6.3 " 16	2,570	3,600
16 " 40	502	1,040
Brighter than 40	14	68

The results in the second column have been obtained by considering the faintest stars, those from 10.0 to 11.0 magnitude. If the class brighter is taken, those stars which appear to be of magnitudes 9.0 to 10.0, we find in a similar way the quantities given in the last column.

There is an increasing divergence between the results. Now it is to be remembered that these figures have been derived from regions at different distances from the sun. Thus the stars which are between sixteen and forty times the brightness of the sun, and which are apparently of magnitude 10 to 11, lie between 398 and 631 parsecs, while those which are apparently of 9.0 to 10.0 magnitude lie between 251 and 398 parsecs.

We may conclude, therefore, that the density of

this class of stars is somewhat less at this greater distance from the sun. Following out this line of reasoning, I have found that the diminution of density of the stars to be as follows:—

Distance	Density	Distance	Density
At 50 parsecs	1'30	At 300 parsecs	0'48
100 „	1'00	400 „	0'32
200 „	0'70	500 „	0'21

Although much weight cannot be attached to the exact figures, one seems justified in saying that there must be a very considerable falling off in the density of the stars between the distances of one hundred and five hundred parsecs. A falling off in the total density of the stars would affect the tables giving the proportion of stars of different brightness, and would increase considerably the proportion of bright stars.

Although the conclusions presented in this paper have been derived from a study of the proper motions of the stars in a small area of the sky, and may be somewhat modified by the investigation of other regions, they may be considered as fairly applicable to the stars in general. The limiting magnitude of the stars that have been considered is nearly 11.0 (on the Potsdam scale), and there are, in the whole sky, half a million stars brighter than this limit of magnitude.

It may be said of them that:—

(i) On the whole, the yellow stars, the stars like the sun in physical conditions, are the nearest.

(ii) They lie within fairly narrow limits of distance—80 per cent. are between one hundred and five hundred parsecs, 10 per cent. nearer than one hundred parsecs, and 10 per cent. further away than five hundred parsecs.

(iii) Going from the yellow to the blue or the orange stars, the average distances increase.

(iv) The red stars are at great distances—an average of about one thousand parsecs.

(v) The stars vary greatly in *intrinsic brightness*. The red stars are specially luminous, being on an average one hundred times as bright as the sun.

(vi) Considering all the stars down to this limit of magnitude, from 90 to 95 per cent. are intrinsically more luminous than the sun.

(vii) When, however, the luminosity of the stars in a given volume of space is considered, there are found to be far more faint than bright stars. There is no contradiction between this conclusion and the last one, because the more distant bright stars are visible, while we only see the faint ones which are comparatively near.

(viii) Evidence has been found that the stars thin out very materially at great distances from the sun.

These conclusions are in harmony with the conception of a finite stellar universe. Most of the stars we see, and a great many fainter ones, are within the distance of one thousand parsecs. Doubtless the stars extend to much greater distances, perhaps ten times as far or further, but we can scarcely doubt that we are near the middle of a finite group of stars, and that the extent of this group is of the order of one thousand to ten thousand parsecs.

### UNIVERSITY AND EDUCATIONAL INTELLIGENCE.

LONDON.—Two lectures on studies in historic magnetism will be given in the autumn by Prof. S. P. Thompson.

The Rogers prize has not been awarded this year. It will be again offered for award in 1916, and the subject of the essay or dissertation will again be "The Nature of Pyrexia and its Relation to Micro-organisms."

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Applications are invited from suitably qualified biologists wishing to engage in research work for the use of the University table at the laboratory of the Marine Biological Association at Plymouth. Preference will be given to members of the University of London. Applications should be sent to the secretary of the Board of Studies in Zoology, the University of London, South Kensington, S.W., and should be accompanied by a statement of the qualifications of the candidate and a brief account of the investigation which he proposes to undertake.

LONDON (UNIVERSITY COLLEGE).—The Drapers' Company has made a grant of 500l. a year for three years, in aid of the work of the department of applied statistics, including the Galton Laboratory of Eugenics and the Drapers' Biometric Laboratory.

THE Russian Imperial Duma has voted in favour of the proposal to establish a faculty of medicine in the University of St. Petersburg.

THE Hele-Shaw prizes in the faculty of engineering of the University of Bristol have been awarded as follows: to Mr. John Rogers (a day student) and Mr. Arthur George Adams (an evening student).

THE trustees of the University of Pennsylvania have sanctioned the admission of women to the medical college of the University. The new regulations will come into force in the autumn of the present year.

ACCORDING to a Reuter telegram honorary degrees have been conferred upon the following members of the British Association by the University of Perth, Western Australia:—Prof. W. Bateson, Prof. Herdman, Dr. A. D. Waller, and Dr. A. C. Haddon.

THE twenty-fifth anniversary of the opening of the Johns Hopkins Hospital will be celebrated in October next. The celebration will begin on October 5 with a meeting to be presided over by Dr. W. H. Welch, at which Prof. Sir William Osler will speak. On October 7 the new Brady Urological Institute will be dedicated.

THE Board of Education has issued [Cd. 7531] the regulations for technical schools, schools of art, and other forms of provision of further education in England and Wales which came into force on August 1. This year the Board has included in the same volume with the regulations for evening schools, day courses in technical institutions and schools of art, the regulations for junior technical schools and those for university tutorial classes. No changes of substance are made in the regulations for junior technical schools or in those for university tutorial classes. Other alterations, which are not numerous, are printed in distinctive type so as to make reference and comparison easy.

THE annual examinations of the National Agricultural Examination Board in the science and practice in dairying will be held for English students on September 12, and following days, at the University College and British Dairy Institute, Reading, and for Scottish students on September 19 and following days at the Dairy School, Kilmarnock. All candidates must have spent at least four months on a dairy farm, and present certificates from approved institutions testifying (1) that he or she has received at least six months' instruction in practical dairy work, and (2) that he or she has attended approved courses in chemistry, bacteriology, and botany, and has satisfied the authorities of the institution of his or her fitness for admission to the examination. Entry forms and all further particulars may be obtained from the Royal Agricultural Society of England, 16 Bedford Square,