THE STARS AROUND THE NORTH POLE.[1]

A KNOWLEDGE of distances of the stars is of fundamental importance in any attempt to describe the stellar universe It is required, before answers can be given to questions on the average distances of stars from one another, their brightness compared with the sun, and the extent to which they reach in space. There are not more than 100 or 150 stars of which the distances have been measured with any degree of accuracy. Although this number is being steadily increased, it is only the stars which are comparatively near to the sun which can be treated individually. For the greater number we have to be content with average values which apply to groups of stars.

A map or a photograph of the stars gives only their bearings—that is to say, their directions as seen from the earth. It gives no information whatever about the distances. One star may be a hundred times as far away as its neighbour on the map. But if two maps are made, separated by a sufficient interval of time, some differences will be found in the relative positions of the stars. These indicate movements either of the stars themselves or of the point from which they are viewed. But the movements which are observed are merely changes of angular position. We cannot tell directly from them either the actual velocities or distances of the stars, but only the ratio between these quantities. It is, however, from the geometrical study of these small angular motions, supplemented by the information obtained from the spectroscope as to the velocities of stars in the line of sight, that our knowledge of their distances is derived.

The problem is in many ways analogous to one which has been completely solved. In the early days of astronomy the movements of the wandering stars or planets were noted. The essential characteristics of the movements were embodied in geometrical formulæ by the Greeks. In the course of time Copernicus showed that these formulæ could be most simply interpreted on the assumption that the earth revolved round the sun. His purely geometrical arguments were, it is true, powerfully reinforced by the revelations of Galileo's telescope. Nevertheless, the planetary system as formulated by Copernicus and Kepler resulted from the observation of the angular movements of the planets and the attempt to give them the simplest possible geometrical interpretation.

Further study of the planetary system has been guided and controlled by the law of gravitation. But the observational data on which our very complete knowledge of the solar system is based, the distances, sizes, and movements of all its members, are a long series of measures of the angular movements as seen from the earth. Linear measurements are only required to obtain the form and dimensions of the earth itself, and thus supply a base line to determine the scale of the system.

The fixed stars present us with a very similar problem. From the study of their small angular movements, supplemented by spectroscopic observations, it is required to construct as far as possible a model of the stellar universe. Such a model would give for each star:—

(i) Its actual position in space, measured along three axes with the sun as origin.

(ii) The velocity in kilometres a second in each of these directions.

(iii) The brightness or luminosity, taking the sun as unit.

(iv) The mass.

(v) The size.

(vi) The physical and chemical constitution.

[1] Discourse delivered at the Royal Institution on Friday, April 24, by Dr. F. W. Dyson, F.R.S.

Of these elements the mass is at present only determinable for double stars, and the size for eclipsing variables. The physical and chemical constitution are known from spectroscopic observations for a considerable number of stars. But the distance and absolute brightness can be found only for a limited number of the nearer stars. Average results can, however, be obtained for the more distant stars, which tell us:—

(1) The number within certain limits of distance from the sun.

(2) The mean velocities of these stars, and what percentage are moving with given velocities, say, for example, between 10 and 20 kilometres a second.

(3) Whether these velocities are irregular or show anything in the nature of streaming in particular directions.

(4) What proportion of the stars are comparable with the sun in intrinsic brightness, and what proportion are ten times or one-tenth as bright, and so on.

Such a description of the stellar system is, to a large extent, within the powers of astronomers, and we nurse the perhaps extravagant hope that generalisations will be discovered which will lead to the formulation of dynamical laws on the constitution of the stellar universe.

A small area round the pole has been chosen as a sample, because this part of the sky has been observed more fully than any other of equal extent. It forms a small cap extending to a distance of 9° from the pole, and covering about 1/160 of the whole sky. In the years 1855–6 Carrington, an English amateur astronomer, well known from his observations of sun-spots, using a very small transit instrument, observed the positions of all the stars in this part of the sky from the brightest down to very faint stars between the 10th and 11th magnitudes. He thus constructed a catalogue, giving with great accuracy the positions of 3700 stars for the year 1855. About the year 1900 these stars were re-observed at Greenwich by a combination of visual and photographic observations. By comparison with the positions as given in Carrington's Catalogue, the angular movement of each of these 3700 stars in forty-five years is determined. These angular movements, or "proper motions" as they are technically called, are the data available for obtaining the actual positions and movements of the stars in space. We have to solve the geometrical problem of making these stars stand out in three dimensions, so that we may see them as we see a picture in a stereoscope.

Now the proper motions of stars are very small. The star of largest proper motion moves only 870″ a century. An idea of the smallness of this motion may be obtained from the fact that it will take two centuries to move a distance equal to the apparent diameter of the sun or moon. There is no star among those near the North Pole with a proper motion so great as this. The following table gives an abstract of the proper motions of the 3726 stars under consideration:—

TABLE I.

Limits of Proper Motion.	Number of Stars.
>40″ a century ...	2
20″—40″ ,, ,, ...	39
10″—20″ ,, ,, ...	134
5″—10″ ,, ,, ...	574
3″— 5″ ,, ,, ...	977
0″— 3″ ,, ,, ...	2,000

It is clear that the stars with large proper motions must either be moving fast or must be comparatively near. These are the alternatives, but for an individual star it is impossible to decide between them.

The table shows how largely the proper motions of stars vary in amount. They differ just as widely in direction. Some signs of irregularity in the directions were first detected by Sir William Herschel, who found that the movement of seven quick-moving stars situated in different parts of the sky were approximately directed to one point. He observed that this would result if the proper motions arose not from the movement of the stars themselves but from that of the point of observation in an opposite direction, and concluded that the solar system was moving towards a point in the constellation Hercules. This conclusion was not universally admitted for some time, but researches by Argelander, Airy, Bessel, and others demonstrated a regular drift among the stars, such as would arise if on their otherwise irregular movements were superposed this common motion. A large number of researches have been made on the exact direction of the sun's motion, and it is now established with some certainty that it is towards a point in right ascension 18h. and declination 35° N., not far in direction from the bright star Vega. The speed of the sun's motion through space has been determined by spectroscopic observations. On the average, stars near Vega appear to be approaching us, stars in the opposite direction to be receding from us. In this way Prof. Campbell has found from the observed velocities of 1500 stars that the solar system is moving at the rate of 19·5 km. a second.

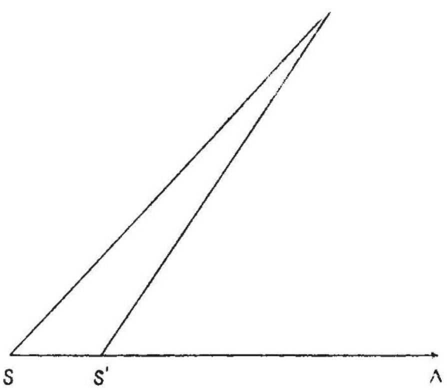


FIG. 1.

The fact that the sun is moving with a velocity of 19·5 km. a second in a known direction supplies us with a means of determining the average distances of groups of stars. This velocity carries the sun forward in a century a distance equal to 412 times the sun's distance from the earth. If at the beginning of the century the sun is at S, and at the end has moved to S′, the angular distance of a star situated at P, and having no motion of its own, will have increased from ASP to AS′P. The difference of these angles, which is the proper motion of the star, is SPS′, and it follows that the distance (SP) can readily be deduced. We cannot, however, say that any individual star is at rest, but if we take a sufficiently large group of stars it is legitimate to suppose that in the average the peculiar movements of the separate stars are eliminated, and the mean distance of the group can be inferred.

During the last twenty or thirty years the proper motions of many stars have been determined by the comparison of modern with earlier observations. Particularly the reduction by Dr. Auwers of Bradley's observations made in 1755 led to the accurate determination of the angular movements of the brighter stars. The proper motions of fainter stars have been found by comparison with observations made in the first half of the nineteenth century. These have all been utilised to determine the direction and angular amount of the drift produced in the stars by the motion of the solar system through space. The results were very puzzling, because different mathematical methods and different groups of stars gave widely different directions for the solar motion. The cause was discovered about ten years ago by Prof. Kapteyn, who found in the proper motions of the stars another indication of regularity, or perhaps it might be called a systematic irregularity smaller than the one discovered by Herschel, but unmistakable when once pointed out. He interpreted these systematic irregularities to mean that the stars are divisible into two groups streaming through one another in opposite directions in space. Prof. Kapteyn's discovery has been submitted to mathematical analysis by Prof. Eddington and Prof. Schwarzschild. Their researches have illuminated the whole subject of stellar motions; and though they are not in entire agreement, they leave no doubt of the existence of a preferential movement among the stars towards the north part of Orion and the diametrically opposite direction in the constellation of the Serpent.

We must next consider the *motus peculiares*—the irregular movements of the stars themselves. From observations of the velocities of stars in the line of sight, especially from those made at the Lick Observatory under Prof. Campbell's direction, it is known that a few stars are moving with great velocities, such as 100 km. a second, while others are moving very slowly. The following analysis of Campbell's results for one class of stars —those of spectral type A—(taken from a paper by Prof. Eddington) shows the proportion of slow-moving, moderate, and quick-moving stars:—

TABLE II.

Velocities	Number of stars observed	Number of stars given by error law
0—5 kil/sec	55	53·4
5—10	47	46·2
10—16	30	38·3
16—25	30	27·4
25—40	10	6·7
>40	0	0

Comparison with the third column of the table shows that the velocities are distributed in accordance with the law of errors. The law is identical with that found by Maxwell for the velocities of the molecules of a gas. In the case of a gas, this distribution of velocities results from the frequent collisions. For the stars there is no evidence that it has resulted from their interaction. It must be regarded as an observational fact which permits us to say that the distribution of the velocities of the stars is stated concisely by this simple mathematical formula.

The three movements—the movement of the solar system in space, the streaming of the stars and their irregular movements are all shown in their proper motions. The figure (taken from a paper by Mr. Jones—Monthly Notices of the R.A.S., vol. lxxiv., p. 196) exhibits the proper motions of some of the brighter stars situated near the North Pole. If the stars had all been placed at the origin they would in a century have spread out as shown in the figure.

This spreading out has been caused by:—

(i) The solar motion, which has shifted the centre of gravity of the swarm towards 180°.

(ii) The peculiar motions of the stars themselves, which have spread them out in the directions towards 90° and 270°.

(iii) The streaming in the direction 0° to 180°, which, combined with the peculiar motions, has made the spreading out much greater in this than in the perpendicular direction. In this part of the sky the streaming happens to be in the direction of and opposite to the solar motion.

Let us now consider the proper motions of the 3700 stars observed by Carrington in the light of these discoveries. The shift of the centre of gravity caused by the solar motion is 1·44″ a century. As we know how far the sun has moved in a century, this gives the average distance of these stars as fifty million times the distance of the sun from the earth. Turning now to the proper motions in a direction perpendicular to that of the sun's motion, which arise from the *motus peculiares* of the stars themselves. Counting these cross proper motions, we find them divided as shown in Table III.

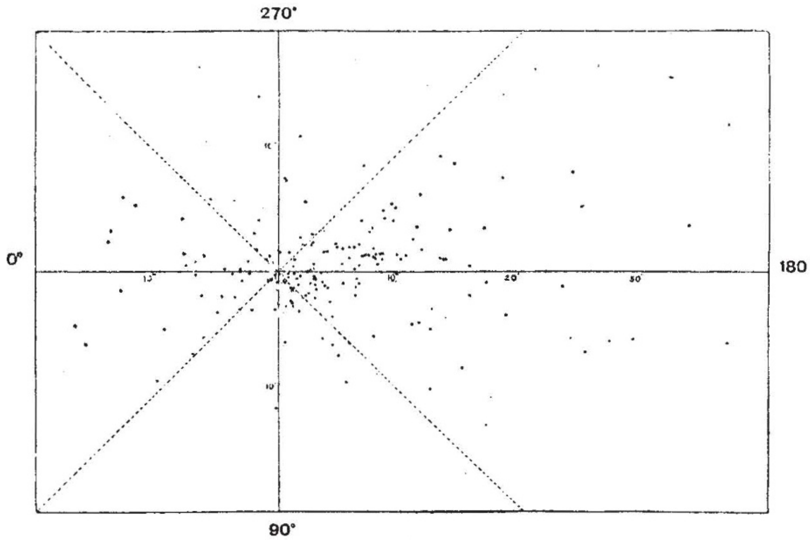


FIG. 2.—Proper Motions of Group A5–F9.

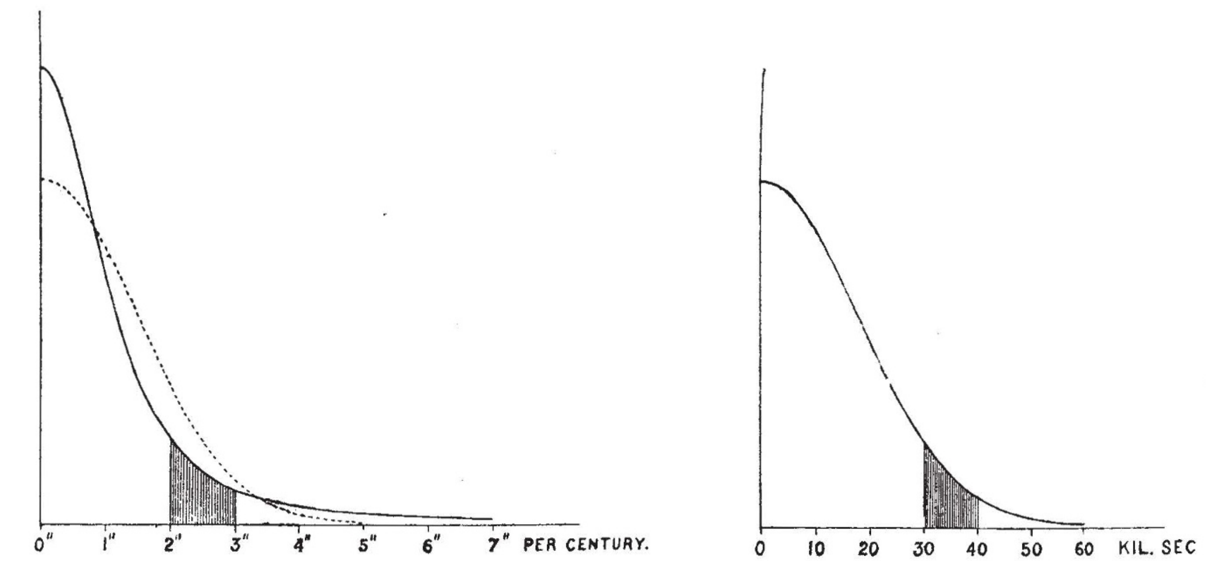
When suitable allowance is made for the accidental error in these observations, it is found that the number less than any given amount τ can be represented by the following algebraical formula:—

$$3700 \frac{\tau}{\sqrt{(\tau^2 + 1{\cdot}47^2)}}.$$

The distribution of the angular velocities is shown in Fig. 3 (A), the total number being represented by the area of the curve; the number, for example, between 2″ and 3″ a century is given by the shaded portion.

Now suppose that all these stars were actually moving with the same velocity, say 10 km. a second, then their distance could be calculated, those with proper motion 1″ a century being forty million times as distant as the sun, those with proper motion 2″ a century twenty million times, those of 4″ a century ten million times, and so on, the larger the proper motion the nearer the star to us.

This is only an illustration; the velocities of the stars are not all the same, but are distributed according to the law of errors. If the distance of each star were known, then by dividing the velocity by the distance the proper motion would be found. We have to find how many are at one distance, how many at another, so that the proper motions will be distributed in accordance with the law found from the observations.



A.—Distribution of angular velocities. FIG. 3. B.—Distribution of linear velocities.

TABLE III.

16 are greater than 15·0″ a century					
25	lie between	10·0″	and	15·0″	a century
33	,,	8·0	,,	10·0	,,
66	,,	6·0	,,	8·0	,,
191	,,	4·0	,,	6·0	,,
873	,,	2·0	,,	4·0	,,
2504	,,	0·0	,,	2·0	,,

Fig. 3 (B) shows the distribution of linear velocities, the shaded portion, for example, giving the proportion moving between 30 and 40 km. a second. Now the distribution of angular velocities is shown in (A), and the question arises: How must the stars be distributed in distance for these two laws to harmonise?

(*To be continued.*)