THURSDAY, NOVEMBER 28, 1912.

SCIENTIFIC WORTHIES.

XXXIX.—Prof. Jules Henri Poincaré, For.Mem.R.S.

It has only happened on one or two occasions that the subject of an article in our series of Scientific Worthies has had to be referred to in the past tense; and we deplore that such should be the case now. Many men of science continued to make important additions to the monument of natural knowledge long after contemporary contributors to this series had paid tribute to their achievements, and fortunately some are still with us. A testimony to good and faithful work has its interest vastly increased when it can be accompanied by the thought that past performances may be equalled, or even excelled, by future accomplishments. This satisfaction is denied us when *Finis* has to be written against a man's work; and though the coral-rock represented by it may be strong and beautiful, it lacks those qualities of activity and growth which were once manifest on its summit and are essential attributes of the scientific spirit.

A great man of science builds not so much for his own generation as for the generations which follow him. As M. Berthelot once said:—"If each of us adds something to the common domain in the field of science, of art, of morality, it is because a long series of generations have lived worked, thought, and suffered before us." For workers of to-day and to-morrow M. Poincaré not only opened new fields, but pointed the way to discovery by those who follow him. Mathematics, physics, astronomy, philosophy, and other domains of intellectual activity have all been extended and illuminated by his genius. The search for truth was for him a passion, and all his work was animated by it. His "Science and Hypothesis" represents an examination into the solidity of the foundations upon which scientific reasoning is based. To the superficial reader the work may appear iconoclastic, but many of the images it destroys should never be set up in the temple of scientific belief; and if they cannot stand before the strong rays of relentless logic, science is better without them. For in nature

> "Beauty is truth, truth beauty; that is all
> Ye know on earth, and all ye need to know."

That such a brilliant and original thinker as M. Poincaré should have died, on July 17 last, at the relatively early age of fifty-eight is a cause of world-wide regret. It would take several articles to do justice to his work and scholarship, but we must here limit ourselves to appreciative mention of a few prominent points of a remarkable career.

M. Poincaré was born at Nancy on April 29, 1854, and commenced his studies at the Lycée there. He afterwards passed successively through l'École Polytechnique and l'École nationale supérieure des Mines, receiving his doctor's degree in mathematical sciences from the University of Paris in 1879. He then began his career as instructor in mathematical analysis at the University of Caen, from which position he was called in 1881 to occupy the chair of physical and experimental mechanics at the Sorbonne (University of Paris). Later he occupied the chair of mathematical physics, and, after the death of M. Tisserand, he passed to that of mathematical astronomy and celestial mechanics. M. Poincaré was elected a member of the Paris Academy of Sciences in 1887, and a member of the French Academy in 1908. He was president of the Academy of Sciences in 1906, and of the Bureau des Longitudes in 1899 and 1909. He was also an honorary member of most of the leading scientific societies of the world, and received honorary degrees from the Universities of Oxford, Cambridge, Glasgow, Christiania, Stockholm, and Brussels. In 1901 the first award of the Sylvester medal of the Royal Society was made to him in recognition of his many and important contributions to mathematical science.

The first volume of a series entitled "Savants du Jour," published in 1909 by Messrs. Gauthier-Villars, of Paris, is devoted to M. Poincaré, and it contains a list of more than four hundred of his publications relating to mathematical analysis, analytical and celestial mechanics, mathematical physics, and philosophy of science. But the value of Poincaré's work is not to be estimated merely by its bulk, although that is unusually large; he never wasted words or wrote on trifles, and his shortest notes, like those of Hermite, are always worth attention. Again, the range of his topics was very wide; arithmetic, probability, function-theory, dynamics, mathematical physics are all indebted to him for results of interest and often of the greatest importance. Finally, he had, in the highest degree, the gift of literary style; few of his scientific compatriots can rival him in directness, simplicity, and grace. There is a story that Clifford, during a walk with a friend, made him understand the gist of Abel's theorem; it is easy to imagine Poincaré, in similar circumstances, successfully expounding the nature of the Fuchsian functions.

Many must be able to recall the delight with which they read those famous memoirs in the early volumes of the *Acta Mathematica,* and the eagerness with which they turned to each new part, in the hope of finding more of this enchanting *causerie.* Few formulæ, and short ones at that; just a succession of brief, almost conversational, sentences opening up a new and vast domain in which even such a subject as elliptic modular functions took a place like that of reciprocants in the general theory of differential invariants; new vistas and new problems presenting themselves on every side. It is easy enough to trace the lineage of the automorphic functions. Immediately suggested by Fuchs's work on differential equations, and actually a generalisation of modular functions, they are historically the outcome of Gauss's memoir on the hypergeometric series, and Riemann's paper on the P-function. To say this is no detraction from Poincaré's merits: the fact is that, like Lejeune-Dirichlet, he won many of his highest triumphs by his extraordinary power of seizing the main points of an existent theory, simplifying it by an appropriate analysis, and then extending it beyond all expectation. Compare, for instance, the present positions of the theories of modular functions and of Fuchsian functions. In the former, apart from further application to arithmetic and the like, the one main problem that still remains is to find out, if possible, the arithmetical characters of all the sub-groups of the modular group; in the latter there are difficulties at the outset, arising from the fact that in certain families of Fuchsian groups there are conditions of inequality which involve troublesome relations connecting the constants of the generating substitutions. In this and in other matters Poincaré did not go into detail: but he pointed out the way for others by his distribution of the functions into families, and by his geometrical method with its non-Euclidean interpretation. Perhaps the crowning result of his work in this direction is his theorem that the coordinates of any point on an algebraic curve can be expressed as one-valued Fuchsian functions of a parameter. This is analogous to the representation of a point on a circle by $(\sin\theta, \cos\theta)$, and is to be distinguished from the Puiseux-Weierstrass representation of an element of the curve.

A more definite example of Poincaré's power of dealing with a classical problem is afforded by his work on rotating fluid masses. Long ago it was shown by Jacobi that an ellipsoid of three unequal axes was a possible figure of relative equilibrium: but it was reserved for Poincaré to take up the problem afresh, and develop the solution into what may fairly be called (apart from details) its final and definite form. He shows the existence of whole families of figures of equilibrium, including as particular cases those already known; gives analytical criteria for stability; and proves that when, by varying the parameter that generates a particular family, we pass from stability to instability, the critical surface is one of "bifurcation," that is, it simultaneously belongs to two distinct families. In some respects this is analogous to the way in which a curve $f(x, y, \mu)=0$, by variation of μ, acquires a double point, and then alters what may be called its connectivity; and in any case, without pressing the analogy, Poincaré's results here seem typical of what happens, with regard to stability, in the variation of dynamical systems. The value and originality of these researches was recognised by Sir G. H. Darwin in his address to the Royal Astronomical Society, when its gold medal was presented to Poincaré (Feb. 9, 1900).

The contributions of Poincaré to celestial mechanics not only brought new life to a subject which showed signs of becoming stale, but undoubtedly opened up a fresh line of investigation. Starting with an idea due to G. W. Hill, who, in his turn, was indebted to Euler, he brought the whole range of his great knowledge and power of analysis to bear on a problem which has baffled the ingenuity of mathematicians for more than two hundred years. That he did not succeed in solving it, either in the old or the modern sense, is no criticism on his achievements; it is sufficient to say that he opened the way and explored a new region by routes which may ultimately lead to the final goal—a demonstration of the stability or instability of the solar system.

His investigations on the general problem of three bodies are principally contained in the three volumes entitled "Les Méthodes Nouvelles de la Mécanique Céleste," which form a natural sequence to the earlier prize essay of 1889. The foundation of the work is the now well-known periodic solution of a set of differential equations. Hill had developed one such solution arising in the motion of the moon round the earth; Poincaré considers periodic solutions of any class of differential equations, examining their general properties and the conditions for their existence. He then takes up the special properties of the equations of dynamics and, descending still further into details, the applications to the problem of three bodies and to restricted cases of this problem. No general method for finding the solutions, nor for discovering the full number of them, is obtained, but these needs are being supplied by the researches of Darwin, Moulton and others into the possible orbits which may be described in various circumstances.

The periodic orbit only represents a particular solution of the equations of motion. Poincaré obtains a general solution within a limited range of the arbitrary constants by considering those differing slightly from the periodic solution. In this connection arise the "characteristic exponents" which may be somewhat loosely taken to give the various periods present in the general solution. These exponents form the bridge which enables him to enter into such questions as the existence of integrals, the analytic forms of possible solutions and the convergence or divergence of the series thus formed. His proof that there cannot exist any algebraic or transcendental integral of the problem of three bodies (under a restriction as to the magnitude of the masses) beyond those known is an important advance on Bruns' result—that no new algebraic integral exists, although the latter is true for any values of the masses.

Not less important is his examination of the older methods from the logical point of view. His presentation of these is nearly always fresh and novel; he is rarely content with previous methods of arriving at the results. This change is perhaps necessary, for he has a different object in view; nevertheless, the reading of them frequently gives the impression that Poincaré simply took the premises and the conclusions and found it less difficult to work out the latter from the former in his own way than to go fully into the author's work. Perhaps the most startling result was his discovery that the majority of the series which have been used to calculate the positions of the bodies of the solar system are divergent. This fact, of course, required an examination into the reasons why the divergent series gave sufficiently accurate results: hence arose the theory of asymptotic series now applied to the representation of many functions.

The crux of the problem is the divergent series. The functions are only represented in the numerical sense by series, and we do not know their limits. Can we argue one way or the other as to the stability of the system? In other words, is the ultimate divergence peculiar to the functions, or is it merely due to our inability to obtain expressions from which a conclusion can be deduced? The question remains unanswered. Gylden believed that he had overcome the difficulty, but Poincaré has shown that it still exists.

Whilst the greater part of Poincaré's researches are thus confined to the logical side of the problems in celestial mechanics, we have occasional papers in which he developed methods useful for actual calculation, in addition to those chapters of the "Méthodes Nouvelles" which are devoted to this part of the subject. Amongst them may be mentioned one on the lunar theory, in which he developed a method with rectangular coordinates which appears to be of value for obtaining algebraic expressions for the coordinates of the moon. There are also two papers dealing with librations in planetary systems which open a way to the more extensive treatment of this complex subject. They have received less notice on account of their narrower range of application; they are incorporated with other matter in his "Leçons de Mécanique Céleste." The recently published volume on cosmogony is of a different nature. It is chiefly a presentation, given originally in a course of lectures, of the works and theories of others, but he does not hesitate to express his own opinions as to their importance in a discussion of the evolution of solar and stellar systems.

A pure mathematician might be pardoned for doubting whether the world, as a whole, benefited by Poincaré's appointment to a chair of mathematical physics. The redactions of his early lectures on electricity and optics have to be read with a certain amount of reserve; he is not yet sure of his ground, and is assimilating the ideas of others. It is difficult to conjecture what he might have done if he had been able to follow up his original bent, which was undoubtedly pure analysis; it would certainly have been something very great. On the other hand, he popularised the Maxwellian theory of electricity, and ultimately mastered it, as well as more recent developments, so that he was able to make contributions to the theory of electrons and that of diffraction. And even in a bare outline, such as this, of his best work, we ought not to pass over his masterly papers on potential and similar subjects, which form the bridge, so to speak, between Neumann and Fredholm.

Poincaré did not disdain to write for a popular audience. "La Science et l'Hypothèse" has deservedly had a wide circulation, and affords a good view of the author's personality. With all his genius, Poincaré was an orthodox thinker by nature; in the case of non-Euclidean geometry, which he fully appreciated, his criticisms are acute and valuable; his sceptical attitude towards Cantor's theory of transfinite numbers is amusing, but not altogether surprising, and is perhaps the only instance of his shutting his eyes to a great mathematical discovery. Kelvin's long opposition to the electromagnetic theory of light is another illustration of the same sort of thing.

To give a just estimate of the value of the researches of Henri Poincaré is not possible at the present time, nor is it necessary. The almost immediate recognition they obtained, the increasing impression of their fundamental importance, and the numbers of students who have followed and expanded the ideas which he laid down with so sure a hand are the best testimony of their worth. We do not know what further contributions he would have made to mathematical science, had he lived, but we do know that what he achieved gives him a permanent place in the history of the subject.

THE PHYSICS OF THE UNIVERSE.

Lehrbuch der kosmischen Physik. By Prof. W. Trabert. Pp. x+662. (Leipzig and Berlin: B. G. Teubner, 1911.)

THE primary justification of a treatise on cosmical physics is to be sought in the principle that economy of communication is of the very essence of science. The author of such a book cannot hope to deal so competently with the individual subjects as the experts to whose writings he must have recourse for his own knowledge, but his work will be a real contribution to the progress of science if he succeeds in imparting unity to his treatment of subjects which have been developed by different workers, each more or less superficial in his knowledge and appreciation of the work of those outside his own branch. Judged from this point of view Prof. Trabert's book is successful. It has been developed according to a definite and well-ordered scheme.

A natural impulse is to compare the book with the masterly treatise with the same title which Arrhenius published ten years ago. The principal difference between the two works is in size and order. The older book covers 1000 pages, of which about 400 are devoted to meteorology; the new one contains 650 pages, of which only about 100 can be spared for meteorology. Arrhenius starts with the "Physik des Himmels," the stars, the sun, the planets, and proceeds from that to the "Physik der Erde," the form and constitution of the earth and the sea, the tides and the ocean currents. He deals finally with the "Physik der Atmosphäre," meteorology, atmospheric electricity, and terrestrial magnetism. Trabert begins with an introductory chapter on the fundamental ideas of the physical concept of the universe. He then deals in order with the form of the earth and its place in the universe, the phenomena of motion —the motion of the sun, the stars and the earth, and tidal and earthquake phenomena, the processes of radiation, with especial reference to the earth's atmosphere, the exchange and transformation of energy, and finally with the development of the universe. Position, motion, energy, result, may be taken to represent briefly the order adopted.

A feature of the book is the care with which the historical development of the principal methods and ideas has been treated, and the retrospective chapters at the end of each section are especially interesting. Thus in the first section the determination of the distances of the sun and moon is traced from its earliest beginnings with Aristarchus and Hipparchus, down to the first exact measurements by Lacaille and Lalande, and the results of Newcomb and Gill. In the second section the different arguments for the rotation of the earth are set forth, including the observed deflection of the wind towards the right; we may commend to those who are sceptical of the effect of the earth's rotation upon motion along the surface the account, on p. 129, of the effect produced on the Hamburg-Harburg railway prior to 1877. In the account of seiches which is given in this section, no mention is made of the work of Chrystal and Wedderburn, and in dealing with star-streams no reference is made to Schwarzschild's hypothesis and the later developments. Such omissions, if they stood alone, might be regarded as incidental to the character of the book, but they indicate a lack of appreciation of recent developments which becomes astonishing when one finds no direct reference to the most important development of Prof. Trabert's own subject in recent years, *i.e.*, the discovery of the stratosphere and its explanation, with the concurrent development of our knowledge of atmospheric radiation and dynamical meteorology.

Apart from this blemish the book appears to be excellent. The use of mathematical formulæ has been avoided as much as possible, but wherever a mathematical demonstration affords the simplest and readiest proof of a result or is necessary for the strict development of the subject, the author has not hesitated to use it; frequently, however, he has given the general outlines of the reasoning in the text, and added the formal proof as a footnote.