at all rounded by the shearing movement of the ice in which they were once embedded, are often scratched and grooved. (See further my paper on the mechanics of glaciers, Q.J.G.S., February, 1883; also NATURE, June 20, 1912.)

I can assure Mr. Reid Moir that the delicate and interesting subject of *patination* presents difficulties to those who (in microscopic and laboratory work) have brought some knowledge of physics and chemistry to bear on the *lithology of the flint*, and that it is not to be dismissed in the easy way he seems to suppose. Nor do I think that even Dr. Sturge (Proc. Prehis. Soc. of E. Anglia) has adequately dealt with the subject or with the possible causes of some phases of "striation." A. IRVING.

Bishop's Stortford, August 22.

THE FIFTH INTERNATIONAL CONGRESS OF MATHEMATICIANS AT CAMBRIDGE.

THE first Mathematical Congress was held at Zurich in 1897, the second at Paris in 1900, the third at Heidelberg in 1904, and the fourth at Rome in 1908. This year's congress met at Cambridge, August 21–28, under the presidency of Sir G. H. Darwin, and was divided into sections as follows:—I, Analysis; II, Geometry; III (*a*), Physical Mathematics; III (*b*), Statistics; IV (*a*), Philosophy and History; IV (*b*), Didactics. Several meetings of the last section were held in connection with the International Commission on the Teaching of Mathematics, which was formed by a resolution of the fourth congress to study and report on the actual state of mathematical teaching in various countries.

Receptions were given by the Chancellor, Lord Rayleigh, in the Fitzwilliam Museum, by Sir G. H. Darwin in St. John's and Christ's Colleges, and by the Master and Fellows of Trinity College. Visits were made to the University observatory and to the works of the Cambridge Scientific Instrument Company. Excursions were arranged to Ely Cathedral, Oxford and Hatfield House. Throughout the week the University and colleges displayed their customary hospitality to the full, and the appreciation of the visitors, both English and foreign, was very evident. The members numbered 572, as compared with 535 at the fourth congress, and included representatives from Brazil, Chile, Egypt, India, Japan, and Mexico. An exhibition organised by the Mathematical Association was arranged in the Cavendish Laboratory, and included English and foreign text-books, examples of school work, models and apparatus, and a most interesting and complete collection of calculating machines. Eight lectures were delivered to the whole congress, and we mention below a few of the less technical points occurring in these, and in the meetings of the didactic section.

Sir G. H. Darwin (Cambridge), in welcoming the congress at the first meeting, referred to the death of Henri Poincaré, whom he described as the one man who alone of all mathematicians might have occupied the position of president of the congress without misgivings as to his fitness. It brought vividly home to him how great a man Poincaré was, when he reflected that, to one incompetent to appreciate fully one half of his work, he yet appeared as a star of the first magnitude.

Prof. E. W. Brown (Yale) lectured on "Periodicity in the Solar System." Newton and his contemporaries aimed at obtaining functions which should express the positions of individual bodies at all epochs. This is now recognised as unattainable; and the position within certain limits of time is expressed by infinite series of terms, some of which are harmonic representing periodic motions, and others expressed as powers of the time representing secular motions. These series are carried to a degree of accuracy exceeding that of the most delicate observation; so that where the calculated positions differ from those observed by a quantity exceeding the possible error of observation, it may be safely assumed that forces are in action other than those postulated in the theory. This is notably the case in the theory of the moon, where the outstanding discrepancy is comparable with the largest of the perturbations due to the planets. Dynamical theory in the case of the asteroids has shown that in the particular case of the problem of four bodies when the mass of one is small, the motion of the latter is unstable for certain ranges of value of the radius vector; and no asteroids have, in fact, been found within these limits. It is possible that an explanation may here be foreshadowed of the dark intervals in Saturn's rings.

Prince B. Galitzin (St. Petersburg) lectured on "The Principles of Instrumental Seismology." The usual seismographic record shows three chief groups of disturbances, due respectively to the longitudinal and transverse waves through the core of the earth, and to the superficial wave round the crust. These, however, are complicated and supplemented by reflections of the deep waves at the surface, and sometimes also by twin earthquakes caused by the primary. The relations between the elastic constants of the core deduced from seismographic observations are in fair agreement with the theory of elasticity of an isotropic medium. But an attempt has been made to construct a more general theory assuming heterogeneity depending on depth. The ideal aim of seismometry must be the determination of the six components of motion of a particle of the earth's crust throughout the whole of a disturbance. Hitherto attention has been confined to the three components of translation. The practical problem of recording the three components of rotation seems to have been solved recently in an apparatus in which induced currents from two pendulums are passed simultaneously in opposite directions through the same galvanometer. There is even reason to believe that the problem of predicting earthquakes is not so hopeless as it would *a priori* seem to be.

Sir W. H. White lectured on "The Place of Mathematics in Engineering Practice." It is matter for surprise that many of the great engineering discoveries of the last century were made by men who had little or no mathematical or

scientific training. On the other hand, much good work was done by French mathematicians in the eighteenth century in laying the foundations of naval architecture. The discussions of recent years have tended to the conclusion that the mathematical portion of an engineer's training is best given in the regular manner by a mathematician, rather than in a selected course by an engineer. There can be no doubt as to the value of mathematics, both in indicating the lines along which experiments must be made and in framing a theory from their results. Many problems, such as that of the design of ship propellers, stand urgently in need of the mathematician's help.

At an extra meeting of Section IV, Mr. P. J. Harding lectured on "The History and Evolution of Arithmetic Division." The two methods of calculation prevalent in Europe previous to the introduction of the Arabic numerals were that of the algorists, who used counting-boards ruled with lines representing successive powers of ten on which counters were placed, and that of the abacus. Arabic numerals followed the trend of commerce from India through Arabia and Italy into northern Europe; so far as we know, they first appeared in Italy in 1202. Subtraction was first performed from the left by scratching out the digits successively, a method evolved from the sand-board used in the East, which was small compared with the size of the numerals, so that successive deletion was necessary. From this followed the method of division by scratching, known as the galleon method owing to a fancied resemblance of the resulting disposition of digits to the form of a ship. The modern method of division first appeared in print in Italy in 1494, but it only superseded the galleon method after a struggle which lasted more than a century. In England its ultimate triumph was largely due to the writing-master Cocker, who advocated it to the exclusion of the older method.

At a special meeting of Section III (*a*), Sir J. J. Thomson (Cambridge) gave a lecture illustrated by experiments on "Multiply Charged Atoms," in which he described some recent investigations on positive charges. He explained the parabolic grouping effected by the simultaneous action of electric and magnetic fields, and showed photographs of the parabolic arcs obtained in various particular cases. In the case of mercury atoms, eight such arcs were obtained, due to one or more of the charges originally carried being lost in transit, so that the particles arrived at the screen with their original energy but with reduced charge.

At an ordinary meeting of Section IV, Dr. A. N. Whitehead (London) read a paper on the principles of mathematics in relation to elementary teaching. The only justification for the inclusion of mathematics in a liberal education is the power of abstraction and deductive reasoning fostered thereby. These powers can only be acquired by constant practice, and no short-cuts are possible. But this does not imply that such powers are to be assumed in the pupil from the outset. On the contrary, no generalisation can be made by the pupil until he is familiar with the raw material from which it is to be made. There is no final degree of rigour in deduction, and the degree to be adopted is a matter for the teacher to decide. His personal choice would be approximately the degree of rigour, though not necessarily the content of Euclid's Elements. No compromise is desirable between the purely utilitarian procedure of looking up a formula in an engineering pocket-book and the acquisition of a mathematical habit of mind by years of practice in abstraction and deduction.

Mr. G. E. St. L. Carson (Tonbridge) read a paper on the place of deduction in elementary mechanics. He suggested that, besides the old method of teaching mechanics in which a structure of deduction was raised on a few postulated laws, and the new method in which principles are demonstrated independently by experiment, there is a third method possible in which the logical interdependence of the principles demonstrated is discussed. Not only is this an aid to understanding the foundations of the subject, but they are shown to constitute a broad inductive basis.

A paper by Dr. T. P. Nunn (London) was read on the proper scope and method of instruction in the calculus in schools. He advocated the teaching of integration by means of graphical illustration on the lines originally adopted by Wallis. This should be followed by a consideration of differentiation as the converse geometrical problem. The teacher should avoid all use of such mystic phrases as "infinite" and "ultimately become," keeping carefully to the definition of limit in terms of finite quantities.

At meetings of Section IV (*b*), in conjunction with the International Commission on the Teaching of Mathematics, reports were presented, with a few explanatory remarks, by delegates from twenty-one countries. The reports exceeded 280 in number, forming an aggregate of more than 9000 octavo pages. These may be obtained from Messrs. Georg et Cie., of Geneva; the English reports have recently been issued in two volumes by the Board of Education. The commission was reappointed for a further period of four years, in order that a digest of these reports may be prepared for the use of teachers in each country. The commission has also conducted special investigations, and reports were presented on the results of two of these.

Prof. C. Runge (Göttingen) presented a report on the mathematical training of the physicist in the university. The need for the closer co-operation of the mathematician and the physicist is strongly felt. It would be of benefit not only to the future physicist or engineer, but also to the student of pure mathematics, if in mathematical lectures theoretical solutions were followed up by numerical computations and applications to material problems. It is also felt that mathematical teaching in the university would be improved if the lecturer were assisted by demonstrators who could keep in personal touch with the student, and aid him as difficulties arise. In com-

menting on this report, Profs. Hobson, Love, and Sir J. Larmor were of opinion that to limit the mathematics of science students to those portions which might be considered of direct utility would destroy that logical unity which is the essential feature of the subject, and relegate it to a subservient position little in keeping with its importance. Sir A. G. Greenhill uttered a warning against the excessive attention engineering pupils are apt to give to descriptive geometry, to the detriment of their studies in the calculus. Sir J. J. Thomson was in favour of physicists learning mathematics from pure mathematicians, if the latter would reserve some of their latest refinements for special lectures.

Prof. D. E. Smith (New York) presented a report on intuition and experiment in mathematical teaching in secondary schools. The object of the inquiry was to ascertain to what extent intuitional methods are at present employed. A general spirit of unrest is apparent. In geometry it may be said that it is the plan of the Teutonic countries to mix the intuitional and deductive work from the outset, while in France, and now in England, the plan is to let an inductive cycle precede a deductive one. The United States is only beginning to talk about the question, whatever tendency there is being towards the Anglo-French plan. The second important movement is the elaboration of the function concept; starting in France within the last twenty years, and vigorously advocated in Germany within the last decade, the movement is, as a whole, too recent to judge of its permanence. A practical form of outdoor mensuration seems to be developing, especially in Austria, Germany, and Switzerland. Geometric drawing and the graphic representation of solids are passing from the hands of the art teacher to the mathematician. Graphic methods of representing functions have become universal in the last generation. The contracted methods of computation that were prominently advocated fifty years ago do not seem to have advanced materially, owing to the feeling that they are not really practical; on the other hand, logarithms have come into general use, and the slide rule is in great favour in technical schools. In general, it may be said that intuitional and experimental methods have made more progress in Austria, Germany, and Switzerland than in England, France, and the United States.

At the final meeting of the congress it was resolved to accept the invitation to Stockholm for the next meeting in 1916. Informal invitations to Budapest and Athens for subsequent meetings were also noted.

THE BRITISH ASSOCIATION AT DUNDEE.

BY the time this issue reaches the reader the British Association will be in full session, and meanwhile there seems to be every prospect of an unusually successful meeting. Dundee is a town of comparatively small population, largely made up of the working classes, but the number of persons resident in the town and neighbourhood who have joined the Association is remarkable. The various towns in which the Association meets are found to differ greatly in this respect, and it occasionally happens that the number of local associates is exceedingly small. Since the year 1901 the Association has held its annual meetings on two occasions abroad and on nine occasions at places within the United Kingdom. The average number of tickets sold at these nine centres before the opening of the reception rooms is 460, and the highest number so sold at any one of the nine was 643; but considerably more than 1100 tickets had already been sold in Dundee by the local committee before the opening of the reception rooms, and by Tuesday evening some 2000 tickets were issued.

This large local addition to the ordinary membership of the Association, together with the unusually large attendance of foreign, American, and Colonial guests, however gratifying it may be to the officers of the Association, renders the task of the local committee a difficult and anxious one. The various halls and Section rooms will be taxed to the utmost, and the various excursions and entertainments will scarcely be sufficient for an attendance so greatly in excess of the estimates that were based on the statistics of recent meetings.

As has already been stated in these columns, the attendance of scientific men from abroad is unusually great, beyond anything indeed that has been seen since the great meeting at Manchester; and this large gathering of foreigners has had its effect in helping to attract the scientific men of our own country. Within the last few days a number of eminent mathematicians, who have attended the recent congress at Cambridge, have made known their intention to be present; geologists are mustering in strength from many countries, tempted to a large extent by the promise of excursions of unusual interest, and a still larger gathering of notable physiologists are coming to do honour to a physiological President.

Every nook and corner of the town is filled almost to overflowing, and members who arrive without having made their arrangements beforehand will have little chance of finding even the simplest houseroom. Private hospitality has provided for between 700 and 800 guests, and every hotel in the town and in the near neighbourhood was filled up many days ago.

It is sometimes said that the British Association is losing ground, but the experience of this meeting shows that the belief is without foundation; not only is the attendance this year fully comparable to the average attendance in the best days of the Association, but there is every prospect also of animated discussion and abundant scientific work. We print this week the inaugural address delivered last night by the president, Prof. E. A. Schäfer, F.R.S., and also the address to be delivered by Prof. H. L. Callendar, F.R.S., before Section A this morning. Other addresses, and reports of the proceedings of the various Sections, will appear in later issues.