better methods of feeding." The importance of sufficient food is emphasised, but the author might have dealt even more fully with this point. As the dairy industry advances, cowkeepers increase their rations, until, in districts where the industry is very highly developed, as in many parts of the home counties, they tend to give too much food for *profitable* milk production.

It has been shown by experiment that an increase in the quantity of the ration leads to an increased production of milk, but the return obtained per unit of food becomes less and less, and finally is not worth as much as the food costs.

The great difficulty about dairy farming in the States seems to be the labour supply. "The special objections raised to the labour on the dairy farm are the long hours, the steady, regular work, and the nature of the work." The same difficulty is felt in British agriculture, and to meet it machinery is being invented. Strenuous efforts are being made to perfect the cow-milking machine, which will considerably ease matters.

E. J. RUSSELL.

ALGEBRAIC NUMBERS.

The Elements of the Theory of Algebraic Numbers. By Prof. L. W. Reid. With an introduction by Prof. D. Hilbert. Pp. xix+454. (New York: The Macmillan Co.; London: Macmillan and Co., Ltd., 1910.) Price 15*s.* net.

IT is almost a misfortune that Gauss and Smith were such consummate masters of mathematical style. Nearly everyone who writes on an arithmetical topic is tempted to imitate them, at least in their brevity and severe deductive method, and the result is that many are frightened away from a delightful study because of its dry and, so to speak, inaccessible aspect.

Prof. Reid's book ought to do much to remove this misapprehension; he has assumed no previous knowledge of the subject beyond elementary arithmetic, and he has been careful to give a very large number of special examples, as well as discussions of special fields. Thus the reader is able to see, much more than is usually possible, the really inductive character of arithmetical research, and is provided with material on which he may make experiments of his own.

As an introduction to the general theory of algebraic numbers, the work follows mainly the treatment of Hilbert and Dedekind; the principal difference from the latter being that, for instance in a quadratic field, an ideal (α, β) is (ultimately) defined as the aggregate of integers $\lambda\alpha+\mu\beta$, where λ, μ are any two integers in the field, and α, β are given integers therein. This saves a good deal of rather delicate reasoning, necessary if Dedekind's own definition of an ideal is adopted, and detracts little, if anything, from the naturalness of the sequence of theorems.

After four chapters dealing with the ordinary rational theory (including the law of quadratic reciprocity), we have four others, each devoted to a special quadratic field, namely, those derived from $\sqrt{-1}$, $\sqrt{-3}$, $\sqrt{2}$, $\sqrt{-5}$ respectively. In the last of these it is made perfectly clear how the law of resolution into prime factors appears to break down, and how it is restored by the introduction of ideals. Moreover, examples are given to show the distinction, in this field, between principal and non-principal ideals.

The next four chapters give general theorems on algebraic numbers, a discussion of the general quadratic field, its discriminant and ideals, and the theory of congruences with respect to ideal moduli in such a field.

Finally, there are two chapters, of a rather more advanced kind, on the units of the general quadratic field, and on the number of its ideal classes. In the latter use is made of Minkowski's remarkable theorem that every ideal class contains an element whose norm does not exceed $|\sqrt{d}|$, where d is the discriminant of the field. Prof. Reid has elsewhere published a list of classes of cubic fields calculated on the same principle; but the work is tentative and laborious, and it is still a desideratum, even for cubic fields, to determine, by some simple method, the fundamental units and representatives of each ideal class. Fortunately, however, in working with an ideal, any one of its forms will do in using it as a modulus, finding its prime factors, and so on: just as (6, 9) or (6, 9, 27) define 3 (as a greatest common measure) just as well as 3 itself for purposes of this kind.

It is very gratifying to see that the higher arithmetic is attracting more and more attention, and it is certain that books like Prof. Reid's will greatly help to popularise "the queen of the sciences," as Gauss so affectionately called it.

G. B. M.

A HISTORY OF EUROPEAN CULTIVATED PLANTS AND DOMESTIC ANIMALS.

Kulturpflanzen und Haustiere in ihrem Uebergang aus Asien nach Griechenland und Italien sowie in das übrige Europa. Historisch-linguistische Skizzen von Victor Hehn. Achte Auflage. Neu herausgegeben von O. Schrader. Mit botanischen Beiträgen von A. Engler und F. Pax. Pp. xxviii+665. (Berlin: Gebrüder Borntraeger, 1911.) Price 17 marks.

VICTOR HEHN'S book, the result of years of labour, first appeared in 1870. A second edition was called for in 1874, to which the essay