Herr Wundt gives results for the year and for the autumn, the semi-diurnal period being most marked in the autumn, for which I find the following harmonic values:—

Height	Harmonic values
1200 m. ...	$A+0\cdot37 \sin(228+x)+0\cdot13 \sin(229+2x)+\&c.$

In this case the amplitudes are in degrees centigrade, and must be multiplied by 1·8 for comparison with the results at Blue Hill. The amplitude of the single diurnal oscillation is nearly the same as the mean between 1000 metres and 1500 metres at Blue Hill, but the phase angle is nearly 180° different. The amplitude of the double diurnal period is a little less than half that found for Blue Hill. However, the method of obtaining the original data was not the same in the two cases.

HENRY HELM CLAYTON.

Readville, Mass., January 8.

A Method of Solving Algebraic Equations.

PROF. RONALD ROSS gave in NATURE of October 29, 1908, an article upon "A Method of solving Algebraic Equations." Without going into the matter itself, or into details concerning it, I beg to state that the above-mentioned process was published in Germany in 1894 in the two following articles by Dr. W. Heymann, professor an der Kgl. Gewerbe-Academie zu Chemnitz in Sachsen:—

(1) Ueber die Auflösung der Gleichungen vom funften Grade (*Zeitschrift für Mathematik und Physik*, xxxix., Jahrgang 1894).

(2) Theorie der An- und Umläufe und Auflösung der Gleichungen vom vierten, fünften und sechsten Grade mittels goniometrischer und hyperbolischer Funktionen (*Journal für die reine und angewandte Mathematik*, cxiii. Band, 1894).

Further publications relating to the same subject, and also by Prof. Heymann, are as follows:—

(3) Ueber die elementare Auflösung transcendenter Gleichungen. Mit Beiträgen zur Ingenieur-Mathematik (*Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*, xxix., Jahrgang 1898).

(4) Ueber Wurzelgruppen, welche durch Umläufe ausgeschnitten werden (*Zeitschrift für Mathematik und Physik*, xlvi. Band, 1901).

I would especially mention, as an article which deals at some length with the geometric explanation of the iteration-process:—

(5) Ueber die Auflösung von Gleichungen durch Iteration auf geometrischer Grundlage (*Jahresbericht*, 1904, *der Techn. Staatslehranstalten zu Chemnitz*).

The author has in this last work thoroughly explained the staircase procession and alternating spiral procession theories, and has also developed the technology of the process, which he further illustrates by a great number of practical examples. I would here direct attention to the fact that this method can be used with advantage in solving transcendant equations. Dr. Heymann has also especially considered in this work those spirals which do not immediately stagnate, but which do so after repeated revolutions; he divides them, therefore, into spirals of the first, second, third . . . *m*th kind.

GEORG SATTLER.

I AM much obliged to Herr Sattler for the information which he has been kind enough to give in regard to my article in NATURE of October 29, 1908, and also for sending me the paper by Prof. Heymann (No. 5) to which he refers. When I wrote my article I could obtain no information concerning previous literature on the method, but since then Mr. W. Stott, secretary of the Liverpool Mathematical Society, has assisted me very greatly with his knowledge of the history of mathematics and with the books in his possession. We are now engaged in making a thorough study of the history of the method, but the following brief account of our progress up to the present may not be out of place.

The method appears to have been discovered by Michael Dary, a gunner in the Tower of London, on August 15, 1674, and was communicated by him in a letter of that date to Isaac Newton (see the "Macclesfield Letters," Correspondence of Scientific Men of the Seventeenth Century, University Press, Oxford, 1841, vol. ii., p. 365). In this letter he indicates clearly that a root of a trinomial equation can be obtained by putting the equation in the form $z^p=az^q+n$, and then by approximating to the value by "iteration"—just as described in my paper. Subsequently he wrote a book called "Interest Epitomised, both Compound and Simple, whereunto is added a Short Appendix for the Solution of Adfected Equations in Numbers by Approachment performed by Logarithms" (London, 1677), but we have not yet been able to procure a copy of this work. Dary was a *protégé* both of Isaac Newton and of Collins. The former subscribed himself in a letter to Dary "your loving friend"; and the latter (to judge by the same "Letters," vol. i., p. 204) tried to advance him, and wrote of him:— "'Tis well known to very many that Mr. Dary hath furnished others with knowledge therein (arithmetic), who, publishing the same, have concealed his name; as, for instance, Dr. John Newton hath lately published a book of Arithmetic, another of Gauging; all that is novel in both he had from Mr. Dary."

I do not know the date when the great Newton first described his method of approximation, but fancy that it must have been done in his "Universal Arithmetic," written about 1669 (the method has been also ascribed to Briggs). The matter is of some interest, because Newton's method is a variant of Dary's—or rather both are special cases of a more general method. In approximating to the intersection of two curves by iteration we may employ either an orthogonal or an oblique geometric construction. The former is the method of Dary (as illustrated in my paper), the latter is the method of Newton, the angle of the oblique construction varying at each step and being taken as that of the tangent of one of the curves at the starting point of the step. Obviously the oblique process gives the quicker approach, and Newton's

$$(x_2=x_1-fx_1/f'x_1)$$

gives the quickest possible if we start sufficiently near the root. Newton was probably aware of this, and consequently did not elaborate Dary's method. Nevertheless, Dary's method is, with certain modifications, the more certain; and, at any step, we can pass from the one process to the other.

The subject now becomes divided into two, the functional theorem, that an iterated function may converge toward the root of an equation, and the converse theorem, that the root of an equation may be calculated by the iteration of a function. The next work which I have seen on the latter theorem is contained in the appendix to the third edition (1830) of Legendre's "Theorie des Nombres" (copies of the second edition may not possess the appendix). He calls this "Méthodes nouvelles pour la Résolution approachée des Équations numeriques," but begins with Newton's method (without acknowledgment) and continues with Dary's. Legendre's paper is curious. He gives the geometric representation of both methods, but omits entirely the "spiral process" mentioned in my paper. We cannot suppose that such a master was ignorant of that process, but must rather believe that he put it aside because he thought it inconvenient for practical calculation (which is not the case if suitable precautions are taken). In order to confine himself to the "staircase process" he puts the proposed equation in the form of "fonctions omales" (homalous), but with the result only that he must often obtain a very slow convergency. In order to extract the successive roots he makes no better suggestion than to divide out the first root already obtained, and the idea of starting the process alternately on the two curves in order to obtain one root after another seems not to have occurred to him. His paper is ingenious, but insufficiently generalised. Prof. Heymann has criticised it to the same effect.

Heymann mentions a number of contributors on the functional side of the theorem, Jakob Bernoulli, Gauss, Jakobi, Stern, Schlömilch, Schröder, Günther, von Schaewen, Hoffmann, Netto, and Isenkrahe. Possibly Babbage, Boole, Galois, and De Morgan may have done as much at an earlier date than some of these writers. De Morgan, in his article on the calculus of functions ("Encyclopedia Metropolitana," London, 1845, vol. ii.,