

with respect to the last paragraph of Prof. Zeeman's article on Prof. Hale's discovery. The magnetic forces indicated by the splitting up of the lines are not sufficient to produce any direct observable magnetic effect at the distance of the earth.

ARTHUR SCHUSTER.

Simla, October 6.

The Magnetic Disturbances of September 29 and Aurora Borealis.

SOME details of an unusually bright aurora, seen at Omaha, U.S.A., on the night of September 28, local time, may be of interest to the readers of NATURE in connection with the three-hour magnetic disturbance recorded on our magnetograms between 4 a.m. and 7 a.m. of September 29, Greenwich time.

The details come from Father Rigge, S.J., director of the Creighton University Observatory, Omaha. The sky was perfectly clear throughout the night. The aurora "seemed to commence suddenly at 9.50 p.m.," September 28, local time, i.e. at 4.15 a.m., September 29, Greenwich time, when the unifilar magnet at Stonyhurst commenced a rapid westward movement up to 62' of arc at 4.40 a.m., returning more slowly in three sudden steps backward at 5.5 a.m., 5.35 a.m., and 6 a.m., accompanied by minor rapid oscillations.

The aurora was watched for two hours, up to the local midnight, and during this time alternations of the scene were observed between brilliant streamers of various lengths and breadths from a well-defined arch, and a broken-up arch accompanied by drifting luminous patches as of fiery clouds. It would have been interesting to compare the times of these changes with the halting movements of the magnetic needle, but the time was recorded only of the first appearance of the streamers, the smaller lengths of which "seemed to come directly out of the ground," and the noted time agrees closely with that of a single break in the first long and rapid deflection of the needle—a short step-back followed by a rush forward to its greatest elongation. The aurora was again looked for at 5 a.m. of the following morning, when nothing was seen in the still unclouded sky.

It is therefore probable that the auroral display began and ended synchronously with this greater deflection of the needle.

The three-hour wave was, then, followed by the usual rapid oscillations consequent upon a magnetic storm until 2.50 p.m., September 29, G.M.T., when another and a greater storm broke out and lasted until 4.30 of the following morning. At Omaha aurora was again seen at 7.15 p.m., September 29, local time, but in a less favourable sky, which clouded over at 9.15, and showed only by the brightened clouds that the aurora was still active at 10 p.m., when the greater oscillations of the magnets were ending.

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Stonyhurst College Observatory, October 21.

A Method of Solving Algebraic Equations.

So far as I can ascertain, the method referred to is not known, at least in its complete form. It is a development of a method described by me in a previous paper ("Verb Functions, with Notes on the Solution of Equations by Operative Division," Proceedings of the Royal Irish Academy, vol. xxv., Sec. A, No. 3, April, 1905), which was reviewed in NATURE of April 25, 1905. I give it here as briefly as possible.

Take, for example, the equation used by Newton to illustrate his method of approximation, namely,

$$x^3 - 2x - 5 = 0,$$

which has one real root, 2.09455. . . . Write the equation in the form $x^3 = 2x + 5$. Select any real number, x_1 ; substitute it for x in the right-hand member of the equation, and then find x_2 from $x_2 = \sqrt[3]{2x_1 + 5}$. Next substitute x_2 for x_1 in the right-hand side of the equation, and find the value of x_3 , and so on. We thus have a series of numbers connected by the equation $x_{n+1}^3 = 2x_n + 5$, and it will be found that whatever number we start with for x_1 , x_n constantly approaches the value of the root. Thus, if we

begin with 11, we have $x_1 = 11$, $x_2 = 3$, $x_3 = 2.2240$, $x_4 = 2.1140$, $x_5 = 2.0975$, $x_6 = 2.0949$, Or, commencing with -100, we obtain $x_1 = -100$, $x_2 = -5.7989$, $x_3 = -1.8756$, $x_4 = 1.0768$, $x_5 = 1.9268$, $x_6 = 2.0688$, $x_7 = 2.0907$, $x_8 = 2.0940$, . . .

Again, take the equation $x^3 - 15x - 4 = 0$, which has three real roots, 4 and $-2 \pm \sqrt{3}$, that is, 4, -0.2678, and -3.7321. Write it in the form $x^3 = 15x + 4$, and begin with any number above the limits of the positive roots, say 16. Substitute this for x in the right side of the equation, and proceed as before. Then $x_1 = 16$, $x_2 = 6.2488$, $x_3 = 4.6062$, $x_4 = 4.0124$, $x_5 = 4.0039$, . . . , which is nearly the first root.

In order to obtain the next lower root take for x_1 a number which is a little less than the first root, say 3.9, and substitute it for x , not in the right side of the equation, but in the left side, so that now $\frac{x_1^3 - 4}{15} = x_2$.

Thus we obtain $x_1 = 3.9$, $x_2 = 3.6880$, $x_3 = 3.0558$, $x_4 = 1.6356$, $x_5 = 0.0351$, $x_6 = -0.2666$, $x_7 = -0.2679$, . . . , which is nearly the second root.

For the third root take a number, say -0.3, which is a little less (algebraically) than the second root, and substitute it for x in the right side of the equation, as done for the first root. We thus obtain $x_1 = -0.3$, $x_2 = -0.7937$, $x_3 = -2.0$, $x_4 = -2.9625$, $x_5 = -3.4313$, $x_6 = -3.6203$, $x_7 = -3.6910$, $x_8 = -3.7169$, $x_9 = -3.7267$, . . . , which is nearly the third root.

We can solve the equation in the same manner by beginning with any number, say -5, which is below the limit of the negative roots, and substituting it for x in the right side of the equation; then after finding the lowest root, substitute a greater number for it in the left side of the equation, and so on. We may thus either descend from the highest to the lowest root, or ascend from the lowest to the highest. It is evident that a root is obtained when $x_{n+1} = x_n$, because the equation is then satisfied.

We took the original equations in the forms $x^3 = 2x + 5$ and $x^3 = 15x + 4$, but we may take them also in the forms $x^2 = 2 + 5/x$ and $x^2 = 15 + 4/x$, or in other forms obtained by ordinary algebraic or operative transformations; and the method of solution is the same.

The rule is most easily explained geometrically. Let $f(x) = 0$ be the original equation. Write it in the form $f_2(x) = f_1(x)$, as may usually be done in many ways. Draw the curves $f_2(x) = y$ and $f_1(x) = y$. Then the roots of $f_2(x) = f_1(x)$ are evidently the abscissæ of the points of intersection of the two curves. The procedure adopted above is really as follows. Select any point, x_1 , on the axis of x , and draw a straight line from it parallel to the axis of y , either in the positive or in the negative direction, until it meets the nearer of the two curves—let us say $f_1(x) = y$. From this second point draw a line parallel to y until it meets $f_2(x) = y$. From the third point draw a line parallel to x until it meets $f_1(x) = y$ again, and from the fourth point one parallel to y until it meets $f_2(x) = y$ again, and so on. Then the abscissa of the first and second points is x_1 , of the third and fourth points is x_2 , of the fifth and sixth points is x_3 , and so on, and x_n must generally approach nearer and nearer to the point of intersection of the two curves—that is, to a root of the original equation.

Fig. 1 represents an intersection where the lines drawn according to the rule all lie within the angles formed by the converging curves. In this case, analytically, x_1, x_2, x_3, \dots , are all either greater or all less than x , the abscissa of the point of intersection, although they constantly approach it. Fig. 2 illustrates the case where the lines ultimately approach the intersection spirally. Here, analytically, x_1, x_2, x_3, \dots alternately oscillate above and below x , although they constantly approach it. The former, or "staircase" procession, occurs while the differential coefficients of the two curves have the same sign; the latter, or alternating "spiral" procession, while they are of opposite signs.

The staircase procession trends in the same direction as the tangent vectors of the curves if $x_2 - x_1$ is positive, and in the opposite direction if $x_2 - x_1$ is negative. A similar law holds for the direction of rotation of the spiral procession. Thus x_1, x_2, x_3, \dots will increase or decrease, either continuously or alternately, according to whether we have taken x_1 on one or the other of the two curves